🔹 UNNS Constants (Current Canon)

1. Limit Ratio Constants

- **Definition:** For a linear UNNS with dominant root α,

$$C_{\text{ratio}} = \lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \alpha.$$

- **Examples:**
  - Fibonacci UNNS → Golden Ratio ϕ.
  - Eisenstein cubic UNNS → ω-related constants.
- **Significance:** Marks the asymptotic growth geometry of each UNNS nest; foundational for embeddings into $\mathbb{Z}[i]$, $\mathbb{Z}[\omega]$, etc.

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2. Gauss–Jacobi–Eisenstein Constants

- **Definition:** Derived from Gauss sums, Jacobi sums, and Eisenstein sums tied to UNNS recurrences modulo primes.
- **Examples:**
  - Quadratic Gauss sum $G(\chi) = \epsilon\sqrt{p}$.
  - Jacobi sum relations: $J(\chi_1,\chi_2) = \frac{G(\chi_1)G(\chi_2)}{G(\chi_1\chi_2)}$.
- **Significance:** Provide explicit arithmetic weights for UNNS lattice embeddings; connect primes, residues, and cyclotomic UNNS layers.

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3. DEC/FEEC Edge Constants (c_1, c_2, C)

- **Definition:** Norm-equivalence and projection constants appearing in the convergence analysis of UNNS → Maxwell mappings.
- **Canonical form:**

$$c_1\|F_h\| \le \|d_h A_h\| \le c_2\|F_h\|, \quad C = \frac{c_2}{c_1}.$$

- **Significance:** Make the stability/convergence of UNNS discretizations explicit. Their numerical value depends on mesh regularity but are structurally **arithmetically bounded by UNNS constants**.

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4. UNNS Paradox Index (UPI)

- **Definition:** A stability threshold measuring symbolic instability:

$$\text{UPI} = \frac{D \cdot R}{M + S},$$

where D=recursion depth, R=self-reference rate, M=morphism divergence, S=memory saturation.

- **Thresholds:**
  - $\text{UPI} < 1$: Safe.
  - $1 \leq \text{UPI} \leq 3$: Transitional.
  - $\text{UPI} > 3$: Unstable.
- **Significance:** Analogous to CFL condition in PDEs; quantifies paradox pressure in recursive substrates.

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5. Gödel Constant

- **Definition:** The inevitability of undecidable statements in recursive nests with $D \geq 2, R > 0$.

$$G(\mathcal{U}) : \exists (P_n) \quad P_n \text{ undecidable in } \mathcal{U}.$$

- **Significance:** Structural invariant of recursion — no UNNS can be paradox-free; incompleteness is a **built-in constant**.

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6. Prime Density Constant (PNT Constant)

- **Definition:** Resonance density of primes in UNNS nests:

$$\pi(x) \sim \frac{x}{\log x}.$$

- **Interpretation in UNNS:** Primes are resonance spikes in recursive substrates; their thinning (density $\sim 1/\log x$) is a universal constant law.
- **Significance:** Encodes prime distribution as an emergent **stability law** of recursion.

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7. Nest Depth Constant (D)

- **Definition:** The minimal number of initial values required for a UNNS to be well-defined.
- **Significance:** A structural invariant of the recurrence (like order in differential equations).

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8. Coefficient Ring Constant (R_{UNNS})

- **Definition:** The smallest algebraic integer ring that contains all coefficients of the UNNS.
- **Examples:** $\mathbb{Z}$, \mathbb Z}[i], \mathbb Z}[\omega].
- **Significance:** Anchors UNNS to cyclotomic/arithmetic lattices.

🔹 Summary

So far, we have **8 constants/invariants** forming the disciplinary backbone of UNNS:

1. **Limit Ratios** (ϕ, ω, α).
2. **Gauss/Jacobi/Eisenstein constants** (cyclotomic sums).
3. **Edge constants** c_1, c_2, C (DEC/FEEC convergence).
4. **Paradox Index (UPI)** (instability threshold).
5. **Gödel Constant** (unavoidable incompleteness).
6. **Prime Density Constant (PNT)** ($x/\log x$ law in UNNS).
7. **Nest Depth Constant** D (recurrence order).
8. **Coefficient Ring Constant** R_{UNNS} (arithmetic anchor).

👉 Together, these move UNNS from "pattern gallery" to a **discipline with invariants, thresholds, and constants**, just like physics has universal constants.