UNNS Paradox Index (UPI): Definition, Properties and Diagnostics

We define and analyze the **UNNS Paradox Index (UPI)**, a scalar diagnostic to flag when Unbounded Nested Number Sequence (UNNS) systems are prone to symbolic instability or paradox. It balances amplifying factors (recursive depth and self-reference) against stabilizing ones (morphism divergence and memory saturation).

Definition:
Let $D \geq 0$ be the recursive depth, $R \in [0,1]$ the self-reference rate, $M \geq 0$ the morphism divergence, and $S \geq 0$ the memory saturation. Then the *UNNS Paradox Index* is defined by $UPI = (D \cdot R) / (M + S)$. A high UPI indicates susceptibility to paradox-prone behavior.

Theorem (Stability Criterion):
Assume error growth is governed by $\varepsilon_{n+1} = \alpha \varepsilon_n + \eta_n$ with bounded forcing $|\eta_n| \leq \eta_\infty$. If $\alpha = UPI < 1$, then ε_n remains bounded with $\limsup \varepsilon_n \leq \eta_\infty / (1-\alpha)$. If $\alpha > 1$, errors grow exponentially unless $\varepsilon_0 = 0$.

Conclusion: Systems are stable when UPI < 1.

Lemma (Monotonicity):
UPI increases with D and R, decreases with M and S. It is homogeneous of degree +1 in (D,R) and degree –1 in (M+S).

Practical thresholds:
Safe: UPI < 1 Caution: 1 ≤ UPI ≤ 3 High risk: UPI > 3

Examples:
1. Fibonacci-like UNNS (D=2, R≈0, M≈1, S=2): UPI=0 → stable.
2. Deep self-referential UNNS (D=10, R=0.5, M=0.5, S=1): UPI≈3.3 → unstable.

Remarks:
- UPI is a diagnostic, not a proof of paradox.
- It is useful for auto-flagging unstable configurations in UNNS explorers.
- Future work: integrate with error norms and FEEC estimates.