The UNNS Observability–Admissibility Duality Theorem

What is observed is not what exists, but what the full operator–constraint stack permits to survive projection.

1 Abstract

We formalize a fundamental structural principle discovered across multiple UNNS Chambers: the existence of substrate-level structure is independent of its observability under a given operator stack. We prove that admissibility operators and observability projections form a duality: admissibility constraints may *reveal* latent structure, while observability projections may *erase* it exactly, without noise or approximation. This theorem resolves apparent contradictions between null experimental results and underlying dynamics, including Bell-type nonviolations and missing geodesic solutions.

2 Preliminaries

Definition 1 (UNNS Substrate)**.** *A* UNNS substrate *is a generative system producing trajectories, fields, or configurations D governed by recursive τ-level dynamics, prior to any observability or admissibility constraints.*

Definition 2 (Admissibility Operator Σ)**.** *An admissibility operator Σ is a constraint-enforcing transformation acting on the substrate space, restricting realizations to those satisfying structural consistency conditions (e.g. conservation, coherence, closure).*

Definition 3 (Observability Projection κ)**.** *An observability projection κ is a measurable mapping from admissible substrate states to observable quantities, including windowing, averaging, thresholding, magnitude-only operations, or coarse-graining.*

Definition 4 (Phase-Exposure Operator Π_ϕ)**.** *A phase-exposure operator Π_ϕ is a pre-κ transformation that lifts latent cyclic degrees of freedom into an explicit, orientation-sensitive representation, preserving phase information under subsequent admissible operations.*

3 The Observability–Admissibility Duality

We consider the composite operator stack:

$$D \xrightarrow{\Sigma} \Sigma(D) \xrightarrow{\Pi_\phi} \Pi_\phi(\Sigma(D)) \xrightarrow{\kappa} \kappa(\Pi_\phi(\Sigma(D))).$$

Each operator layer may alter which structural properties remain accessible.

4 Main Result

Theorem 1 (UNNS Observability–Admissibility Duality Theorem)**.** *Let D be a UNNS substrate realization containing a structural property P (e.g. phase coupling, nonseparability, geodesic optimality). Then:*

1. *There exist admissibility operators Σ such that*

$$P \notin \mathrm{Obs}(D) \quad \textit{but} \quad P \in \mathrm{Obs}(\Sigma(D)).$$

2. *There exist observability projections κ such that*

$$P \in \mathrm{Obs}(D) \quad \textit{but} \quad P \notin \mathrm{Obs}(\kappa(D)).$$

3. *The erasure in (2) can be exact and deterministic, even in the absence of noise, finite sampling, or approximation error.*

4. *The revelation in (1) can exhibit sharp threshold behavior with respect to admissibility parameters, producing discontinuous transitions in observability.*

5. *Therefore, absence of an observable signature of P under any fixed operator stack implies neither absence nor weakness of P at the substrate level.*

5 Proof Sketch

Proof sketch. Admissibility operators Σ restrict the realization space, eliminating configurations that violate structural constraints. This restriction can collapse diffuse or inconsistent substrate realizations into sharply defined structural solutions, rendering P observable.

Conversely, observability projections κ may quotient out degrees of freedom essential to P (e.g. orientation, phase sign, relational alignment). When κ is invariant under a symmetry that distinguishes realizations supporting P, the observable image necessarily collapses, erasing P exactly.

Both effects are operator-algebraic and independent of stochasticity. Since Σ and κ act in opposite logical directions (constraint vs. projection), they form a dual pair with respect to structural observability. □

Theorem 2 (UNNS Observability–Admissibility Complementarity Principle)**.** *Let D be a UNNS substrate realization possessing a latent structural property P (e.g. phase coupling, nonseparability, geodesic optimality). Then there exist two dual classes of operators:*

1. *observability-modifying operators Π (e.g. phase exposure or erasure),*

2. *admissibility-modifying operators Σ (constraint gating),*

such that:

1. *P may be* rendered observable or unobservable *by Π without altering D,*

2. *P may be* rendered admissible or inadmissible *by Σ without altering D,*

3. *both effects can occur via sharp threshold transitions in operator parameters,*

4. *and neither effect requires noise, approximation, stochasticity, or finite-sample error.*

Therefore, structural accessibility in UNNS is neither monotonic nor absolute, but jointly determined by the ordered pair (Σ, Π) acting on D.

6 Cross-Chamber Instantiation: XL XXXI

We now demonstrate a concrete realization of the Observability–Admissibility Complementarity Principle via two independent UNNS chambers operating on distinct substrates and metrics.

6.1 Chamber XL: Phase-Erasure Hides Structure

Substrate. Phase-coupled τ-fields with fixed latent offset $\delta = \pi/8$.

Observability regimes.

- Phase-exposed channel (Π_ϕ):

$$|S| \approx 2.61 \quad \text{(nonseparable)}$$

- Phase-erased channel (κ):

$$|S| \approx 0.71 \quad \text{(separable)}$$

Mechanism. Magnitude-only observables invariant under $\phi \mapsto \phi + \pi$ destroy phase orientation, collapsing Bell violations despite unchanged substrate dynamics.

Verdict. *ERASURE ARTIFACT*: nonseparability exists but is operator-dependent.

6.2 Chamber XXXI: Σ-Gating Reveals Structure

Substrate. τ-field trajectory optimization under admissibility gating.

Admissibility regimes.

- $\sigma = 0$ (no gating):

$$\text{minD} \approx 4.5 \times 10^{-4}, \quad \text{no geodesics}$$

- $\sigma \geq 0.02$:

$$\text{minD} = 0, \quad \text{stable geodesics}$$

Mechanism. Σ-gating enforces admissibility constraints that collapse diffuse solution space into exact geodesic realizations.

Verdict. *REVELATION ARTIFACT*: structure becomes accessible only after constraint enforcement.

6.3 Quantitative Alignment

Metric	Chamber XL	Chamber XXXI
Transition magnitude	$\Delta S \approx 1.91$ (72.9%)	ΔminD = 100%
Direction	structure $\to$ hidden	structure $\to$ revealed
Signature	$2.61 \to 0.71$	$4.5 \times 10^{-4} \to 0$
Robustness	$p = 0.0099$	93.8% admissible

Both transitions are sharp, operator-induced, and non-gradual.

6.4 Unified Operator Interpretation

The two chambers instantiate dual operator effects:

	Phase Exposure (Π_ϕ)	Σ-Gating
Role	observability lifting	admissibility enforcement
Effect	prevents κ-erasure	enables admissible solutions
Failure mode	Bell nonviolation	geodesic absence

Thus, both chambers confirm:

> Absence of signal does not imply absence of structure, but reflects interface-specific accessibility.

7 Corollaries

Corollary 1 (Null Result Non-Equivalence)**.** *A null observable result under κ does not imply absence of substrate structure; it diagnoses only the information-losing properties of the operator stack.*

Corollary 2 (Operator-Relative Properties)**.** *Structural predicates such as "entangled", "separable", "geodesic", or "optimal" are not absolute properties of D, but of the tuple $(D, \Sigma, \Pi_\phi, \kappa)$.*

8 Implications

This theorem explains:

- Bell-type nonviolations under phase-erasing observables despite underlying coupling,
- Emergence of perfect geodesics only after admissibility gating,
- Systematic failure of naive searches for structure in unconstrained substrates,
- The reproducibility of sharp observability thresholds across unrelated chambers.

9 Formal Counterposition to Bell–Local Realism

Bell's theorem constrains theories satisfying *local realism*, defined by the joint assumptions that:

1. outcomes are determined by setting-independent latent variables,
2. measurement outcomes factorize across space-like separated contexts,
3. observables are complete with respect to the relevant degrees of freedom.

The UNNS framework rejects the third assumption while leaving the first two explicitly testable.

9.1 Key Distinction

Bell inequalities constrain probability distributions of the form

$$\mathbb{P}(A, B \mid \alpha, \beta) = \int \mathbb{P}(A \mid \alpha, \Lambda)\mathbb{P}(B \mid \beta, \Lambda)\, d\mu(\Lambda),$$

given a fixed observable family.

UNNS makes no claim that all observable families preserve the information required to test such factorizations.

9.2 Observability Relativity

Let κ be an observable projection. Define Bell-locality relative to κ as:

$$\mathbb{P}_\kappa(A, B \mid \alpha, \beta) = \int \mathbb{P}_\kappa(A \mid \alpha, \Lambda)\mathbb{P}_\kappa(B \mid \beta, \Lambda)\, d\mu(\Lambda).$$

UNNS explicitly allows:

$$\mathbb{P}_{\Pi_\phi}(A, B \mid \alpha, \beta) \text{ non-factorizable} \quad \text{while} \quad \mathbb{P}_\kappa(A, B \mid \alpha, \beta) \text{ factorizable.}$$

This does not violate Bell's theorem; it instantiates different observable algebras.

9.3 Why Bell Violations Can Disappear

Bell tests assume that the measured observables are:

- complete with respect to the relevant hidden degrees of freedom,
- invariant under admissible experimental contexts,
- not systematically erasing relational structure.

Chamber XL demonstrates that magnitude-only or windowed observables violate these assumptions by construction.

Thus, Bell nonviolation under κ does *not* imply:

- separability of the substrate,
- absence of coupling,
- validity of local realism at the substrate level.

[DI-QKD Failure Mode: Erasure-Induced Fake Security] Let a two-wing UNNS substrate generate paired data $D = (D^A, D^B)$ with a latent coupling structure that is *nonseparable* under at least one admissible exposure channel, i.e. there exists an admissible Π_ϕ and a (possibly simple) κ_{DI} such that

$$\mathsf{Chan}_{\mathrm{cert}} := \kappa_{\mathrm{DI}} \circ \Pi_\phi \circ \Sigma \quad \text{yields} \quad |S_{\mathrm{cert}}| > 2$$

on the Σ-admissible subset, passing the robustness protocol $\mathcal{R}$.

Assume, however, that the *actual* key bits are produced by an alternative channel

$$\mathsf{Chan}_{\mathrm{key}} := \kappa_{\mathrm{key}} \circ \Pi \circ \Sigma$$

whose effective action lies in an erasure class $\mathcal{E}$ (phase-erasing / relationally-erasing), so that the induced output statistics are Bell-local:

$$|S_{\mathrm{key}}| \leq 2 \quad \text{(within statistical tolerance)},$$

even though the underlying substrate admits nonseparability under $\mathsf{Chan}_{\mathrm{cert}}$.

Then there exists an adversarial (or simply mismatched) device realization consistent with the observed Bell-local outputs under $\mathsf{Chan}_{\mathrm{key}}$ for which the apparent DI-QKD security claim is *spurious*: the outputs can be explained by a Bell-local model at the level of the reported bits, and no device-independent secrecy guarantee follows from the observed data.

Equivalently: *Bell-local outputs under an erasing measurement channel admit "fake security" even when latent coupling exists*, because the erasure map can destroy precisely the relational degree of freedom that would otherwise certify min-entropy via a Bell violation.

Proof sketch. Device-independent secrecy bounds are derived from constraints on the *observed* input–output statistics, typically requiring a Bell violation for the same outputs that will be used as raw key (or for a statistically coupled sample from the identical channel). If $\mathsf{Chan}_{\text{key}} \in \mathcal{E}$, then distinct substrate regimes (including those with phase-locked or otherwise nonseparable latent structure) can be mapped to Bell-local statistics at the output. Therefore, the empirical condition needed to lower-bound secrecy (a Bell violation for the key-generation channel) is absent, and a Bell-local explanation remains consistent with the reported data. Hence no DI guarantee can be inferred, regardless of latent coupling detectable under a different channel $\mathsf{Chan}_{\text{cert}}$. □

It diagnoses an observability erasure channel.

9.4 UNNS-Compatible Reformulation of Bell Tests

UNNS reframes Bell tests as conditional statements:

> Given an admissibility regime Σ and observable family κ, nonviolation implies separability *relative to* (Σ, κ).

Phase exposure (Π_ϕ) alters the observable algebra and restores access to correlations already present in the dynamics.

9.5 Conclusion

Bell's theorem constrains *models of observables.* UNNS demonstrates that observables are operator-relative, and that changing the operator stack can convert Bell-local statistics into Bell-nonlocal statistics without altering the substrate.

This places Bell violations in their proper role: *witnesses of interface adequacy, not ontological arbiters.*

10 Device-Independent QKD Corollary (UNNS Form)

Definition 5 (DI-QKD operational target)**.** *A device-independent QKD protocol aims to certify secret key generation from observed input–output statistics alone, without trusting the internal mechanism of the devices. Operationally, the certification hinges on: (i) a Bell/CHSH violation in the* implemented *measurement channel, (ii) a no-signalling / causal separation condition between the two wings during each trial, and (iii) an i.i.d. or controlled-memory assumption (or an explicit de-Finetti / entropy-accumulation style substitute).*

Definition 6 (UNNS measurement channel and erasure class)**.** *Let* Chan *denote the effective measurement channel implemented by the operator stack used to produce the reported bits:*

$$\mathsf{Chan} := \kappa \circ \Pi \circ \Sigma,$$

where Σ is the admissibility gate, Π is any pre-κ exposure operator (e.g. Π_ϕ), and κ is the coarse observable projection producing the bits.

Define an erasure class $\mathcal{E}$ *as a family of channels* Chan *that are invariant under a phase flip (or more generally, erase a relational degree of freedom), e.g. $\phi \mapsto \phi + \pi$, so that latent nonseparability can be mapped into Bell-local statistics.*

Corollary 3 (UNNS DI-QKD Admissible-Channel Corollary)**.** *Assume a two-wing UNNS substrate produces paired data $D = (D^A, D^B)$ and that the protocol extracts raw key bits via an operator stack* $\mathsf{Chan} = \kappa \circ \Pi \circ \Sigma$.

Then:

1. ***(Necessity of non-erasing certification channel).*** *If* $\mathsf{Chan} \in \mathcal{E}$ *(phase-erasing or relationally-erasing class), then a null CHSH result $|S| \leq 2$ under* Chan cannot *certify either substrate separability or DI-security. It only certifies that the* reported *statistics are Bell-local relative to an erasing channel.*

2. ***(Sufficient interface condition for DI-style certification).*** *If there exists an admissible exposure operator Π_ϕ such that the* admissible *channel*

$$\mathsf{Chan}_{\mathrm{cert}} := \kappa_{\mathrm{DI}} \circ \Pi_\phi \circ \Sigma$$

   yields a statistically significant violation $|S_{\mathrm{cert}}| > 2$ on the Σ-admissible subset, and *the robustness protocol $\mathcal{R}$ (stride sweep, time-shift null, surrogate tests, and memory controls) passes, then the observed bits are generated by a channel whose correlations are non-factorizable within the admissible operator family.*

   In that case, any DI-QKD claim must be formulated relative to $\mathsf{Chan}_{\mathrm{cert}}$*:*

$$\textit{DI-security claim} \Rightarrow \textit{security of bits produced under } \mathsf{Chan}_{\mathrm{cert}}, \textit{ not under arbitrary } \mathsf{Chan}.$$

3. ***(Key practical implication).*** *In UNNS, DI-QKD security cannot be inferred from the existence of latent coupling alone; it requires that the* key-extraction channel itself *is non-erasing. Equivalently: DI-QKD is an* interface property *(channel-relative), not a substrate property.*

Proof sketch. DI-QKD relies on a Bell-violation-based entropy guarantee for the *reported* outputs. If the reporting channel lies in an erasure class $\mathcal{E}$, then distinct substrate states (including nonseparable ones) can map to Bell-local statistics, so $|S| \leq 2$ cannot distinguish secure-from-insecure regimes. Conversely, if an admissible, non-erasing certification channel produces $|S| > 2$ robustly under $\mathcal{R}$, then the reported outputs exhibit operator-relative non-factorizability, satisfying the necessary correlation precondition used by DI-QKD security analyses. The restriction to the admissible subset is enforced by Σ, matching the empirical chamber practice. □

Interpretation (link to Chambers XL and XXXI). Chamber XL supplies an explicit example where Π_ϕ yields $|S| > 2$ while κ-erasure yields $|S| \leq 2$, demonstrating item (1). Chamber XXXI supplies the admissibility mechanism (Σ-gating) required to ensure that the certification channel and robustness checks are defined on the valid (admissible) region, supporting item (2).

11 Conclusion

The UNNS Observability–Admissibility Duality Theorem establishes that observability is neither a proxy for existence nor a monotonic function of measurement fidelity. Instead, it is a structured outcome of operator composition. This reframes null results as diagnostic tools for operator design rather than evidence against substrate structure.

A Methods Appendix: Empirical Grounding in Chambers XL and XXXI

This appendix specifies the concrete experimental procedures and datasets used to instantiate the Observability–Admissibility Duality Theorem.

A.1 Chamber XL: Phase-Exposure vs. Phase-Erasure

Dataset. Chamber XL operates on paired τ-field trajectories generated from a common UNNS substrate run. Each trial produces two wings $(\tau_t^A, \tau_t^B)_{t=0}^T$ with a fixed latent phase offset $\delta = \pi/8$ in the coupled condition, and δ randomized in the independent control condition.

The datasets used include:

- `ChamberXL_v1.2_synthetic_coupled_2026-01-26.json`
- `ChamberXL_v1.2_synthetic_independent_2026-01-26.zip`

Operators. Phase exposure is implemented via Π_ϕ using a gradient-angle proxy on the τ-field:

$$\phi_t(x, y) := \operatorname{atan2}(\partial_y \tau_t, \partial_x \tau_t).$$

Phase erasure is implemented by magnitude-only observables invariant under $\phi \mapsto \phi + \pi$, such as $|\cos(\phi - \theta)|$.

Witness. Nonseparability is evaluated using the CHSH functional

$$S = E(\alpha, \beta) + E(\alpha, \beta') + E(\alpha', \beta) - E(\alpha', \beta'),$$

with standard angle choices and admissibility gating inherited from Chamber XIV.

Observed Regimes.

- Phase-exposed: $|S| \approx 2.61$ (statistically significant violation)
- Phase-erased: $|S| \approx 0.71$ (strict separable regime)

This constitutes an explicit instantiation of exact observability erasure without noise, sampling bias, or locality violation.

A.2 Chamber XXXI: Σ-Gated Geodesic Emergence

Dataset. Chamber XXXI analyzes trajectory optimization under a tunable admissibility parameter σ controlling Σ-gating strength. Key datasets include:

- `chamber_xxxi_v1.0.5_sigma-sweep_m1_2026-01-26.json`
- `chamber_xxxi_v1.0.5_toy_example_2026-01-26.json`

Metrics. Structural accessibility is measured via:

- minimum divergence minD from ideal geodesic cost,
- physical geodesic count (normalized),
- robustness across σ sweeps.

Observed Transition.

$$\sigma = 0 \;\Rightarrow\; \mathrm{minD} \approx 4.5 \times 10^{-4}, \quad \text{no geodesics}$$

$$\sigma \geq 0.02 \;\Rightarrow\; \mathrm{minD} = 0, \quad \text{stable geodesics}$$

The transition is sharp and persists across 93.8% of admissible configurations.

A.3 Cross-Chamber Alignment

Chamber XL demonstrates that κ-level projections can *destroy* access to real substrate structure, while Chamber XXXI demonstrates that Σ-level constraints can *enable* access to real substrate structure.

Together they instantiate the dual clauses of the main theorem.