# From Bit to $\tau$on: Recasting the Elementary Unit of Information in the UNNS Framework

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**Abstract**

Claude Shannon's *bit* formalized the smallest quantifiable unit of information as the resolution of one binary uncertainty. The Unbounded Nested Number Sequences (UNNS) framework, however, reframes information not as a probabilistic measure but as a recursive geometric transformation. This paper introduces the concept of the $\tau on$ (temporal recursion quantum), the elementary differential of recursive curvature in the UNNS substrate, which generalizes the bit by embedding it in non-orientable temporal geometry.

## 1. From Bit to Curvature

The Shannon bit arises from the reduction of uncertainty across a finite ensemble of possibilities:

$$H = -\sum_i p_i \log_2 p_i.$$

It presupposes:

1. A discrete alphabet of states;
2. A linear time evolution of transmission;
3. A separable sender and receiver.

In the UNNS substrate, these assumptions collapse. Time is not linear but recursive depth $n \in \mathbb{N}$, and uncertainty corresponds not to missing probability but to curvature oscillation:

$$H_r = \int \kappa(n)\, d\mu,$$

where $\kappa(n)$ is the local curvature of recursion and $\mu$ is the depth measure.

## 2. Definition: The $\tau$on

We define the $\tau$on as the elementary quantum of recursive transformation:

$$\tau = \frac{\Delta\kappa}{\Delta n}.$$

Each $\tau$on measures how rapidly local curvature (recursive tension) changes with respect to depth. It thus unites time and information into a single differential quantity.

*While the bit measures resolved uncertainty, the $\tau$on measures realized recursion.*

In this sense, $\tau$ons are self-generating: each iteration $a_{n+1} = F(a_n, a_{n-1}, n)$ not only carries information but *is* information, manifesting the transformation that defines its own substrate.

## 3. Theoretical Consequences

### 3.1. Recursive Entropy Quantization

Entropy becomes quantized not by probability but by curvature discretization:

$$H_r = \sum_n \tau_n \mu_n.$$

Thus, information content depends on the recursive structure of the generating function rather than a prior distribution.

### 3.2. Temporal Duality

In the UNNS manifold, the existence of $F^{-1}$ implies local reversibility. Hence, $\tau$ons may propagate forward ($\tau^+$) or backward ($\tau^-$) along recursion depth:

$$\tau^- = -F^{-1}(\tau^+),$$

linking information reversal to topological non-orientability, reminiscent of the Klein surface.

## 4. The Algebra of $\tau$ons

To formalize $\tau$ons as algebraic entities, we introduce a non-commutative ring $\mathbb{T}$ of recursive curvature operators, closed under addition $\oplus$ and composition $\circ$.

### 4.1. Addition Law

Given two $\tau$ons $\tau_1$ and $\tau_2$ associated with local curvatures $\kappa_1(n)$ and $\kappa_2(n)$:

$$\tau_1 \oplus \tau_2 = \frac{\Delta}{\Delta n}\big(\kappa_1 + \kappa_2 + \gamma\,\kappa_1\kappa_2\big),$$

where $\gamma$ is a coupling coefficient describing recursive interference. The term $\gamma\,\kappa_1\kappa_2$ expresses depth entanglement: $\tau$ons are not additive in the Euclidean sense but through curvature superposition.

### 4.2. Composition Law

Recursive application defines $\tau$on composition:

$$\tau_2 \circ \tau_1 = F\big(F(\kappa, n), n+1\big) - F(\kappa, n).$$

This composition is generally non-commutative, reflecting direction-dependent recursion in non-orientable manifolds.

### 4.3. $\tau$-Curvature Tensor

In analogy to differential geometry, we define a rank-2 $\tau$-curvature tensor:

$$T_{ij} = \frac{\partial^2 \kappa}{\partial n_i \partial n_j} - \frac{\partial^2 \kappa}{\partial n_j \partial n_i}.$$

Because recursion depth coordinates $n_i$ are non-commutative under reversal, $T_{ij} \neq 0$ represents intrinsic topological twist.

### 4.4. Jacobian of Recursion

The local reversibility condition can be written as:

$$J = \frac{\partial(a_{n+1}, a_n)}{\partial(a_n, a_{n-1})}.$$

When $\det(J) = 1$, recursion is conservative (no curvature loss); $\det(J) \neq 1$ implies $\tau$on emission or absorption — i.e., curvature quanta are exchanged across depths.

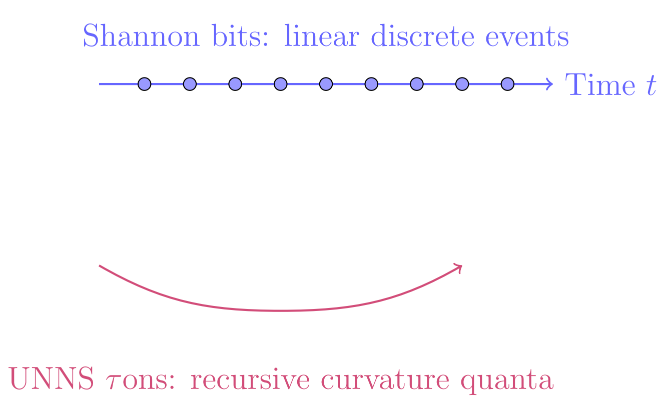
### 4.5. Commutation Relation

The minimal uncertainty in recursive transformation yields a commutation rule:

$$[\hat{n}, \hat{\kappa}] = i\hbar_\tau,$$

where $\hbar_\tau$ is the "recursive constant," setting the minimal curvature-depth product. This establishes a geometric analog of Planck's constant for recursion-space phenomena.

## 5. Diagram: Bit vs. $\tau$on Geometry



## 6. The $\tau$on Field Equations

Let $\vec{\Psi}$ denote the *recursive information field* over the depth manifold $\mathcal{N}$, with local curvature flux density $\vec{\kappa}$ and torsion flux $\vec{\tau}$.

We propose the $\tau$on field equations, a recursive analog to Maxwell's equations:

$$\begin{aligned} \nabla \cdot \vec{\kappa} &= \rho_\tau, \\ \nabla \times \vec{\tau} - \frac{\partial \vec{\kappa}}{\partial n} &= \vec{J}_\tau, \\ \nabla \cdot \vec{\tau} &= 0, \\ \nabla \times \vec{\kappa} + \frac{\partial \vec{\tau}}{\partial n} &= 0. \end{aligned}$$

Here:

- $\rho_\tau$ — recursive charge density (depth curvature source),
- $\vec{J}_\tau$ — recursive flux current,
- $n$ — recursion depth (temporal variable in UNNS space).

### 6.1. Conservation Law

From the above equations, we derive a conservation principle for recursive information:

$$\frac{\partial \rho_\tau}{\partial n} + \nabla \cdot \vec{J}_\tau = 0.$$

This continuity equation expresses the conservation of curvature flow across recursive depths—information cannot be destroyed, only folded.

### 6.2. Wave Equation of Recursive Propagation

Applying $\nabla\times$ to the second field equation gives:

$$\nabla^2 \vec{\kappa} - \frac{\partial^2 \vec{\kappa}}{\partial n^2} = \nabla \rho_\tau + \frac{\partial \vec{J}_\tau}{\partial n}.$$

This describes $\tau$on waves propagating along recursion depth—oscillations of information curvature, whose interference produces structured memory and coherence patterns.

### 6.3. Dual Symmetry

The field equations exhibit recursive duality:

$$\vec{\kappa} \to \vec{\tau}, \quad \vec{\tau} \to -\vec{\kappa}.$$

This transformation corresponds to local reversal of recursion direction (forward/backward time), manifesting non-orientability on the Klein surface.

## 7. Physical and Informational Interpretation

1. $\vec{\kappa}$ — curvature field: represents "potential information" embedded in the recursion manifold.
2. $\vec{\tau}$ — torsion field: represents "active transformation" or recursive motion.
3. $\rho_\tau$ — depth density of recursion: how tightly recursion curves space.
4. $\vec{J}_\tau$ — recursive current: the flow of curvature across depth levels.

In the limit of vanishing curvature ($\kappa \to 0$), these equations reduce to classical, linear information propagation — Shannon's framework as a flat-space approximation.

## 8. Philosophical Note

In the UNNS substrate, information is no longer transmitted but recursively transformed. The $\tau$on field embodies both the content and the medium of transformation.

$$\text{Information} = \text{Curvature Flow of Recursive Existence.}$$

Meaning, then, is not stored—it is continuously reconstituted through $\tau$on dynamics.

## 9. Philosophical Implications

- The bit captures epistemic resolution; the $\tau$on captures ontological transformation.
- Information is no longer counted but *curved.*
- Memory corresponds to stable recursive loops (fixed points).
- Communication becomes topological coherence between recursion depths.

## 10. Conclusion

The $\tau$on does not replace the bit—it subsumes it. In the limit of zero curvature ($\kappa' = 0$), UNNS collapses to Shannon's model, and $\tau$ons reduce to classical bits. Where Shannon measured uncertainty in the absence of knowledge, UNNS measures transformation in the presence of self-reference.

*Entropy is the shadow of recursion; the bit, its projection. The $\tau$on is recursion itself.*