Recursive Field Unification: τon Dynamics, Entanglement Entropy, and the Klein Manifold Geometry of Information

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Abstract

This paper extends the τon field theory of recursive information to a unified geometric model incorporating entanglement entropy and non-orientable topology. The Unbounded Nested Number Sequences (UNNS) substrate is treated as a non-orientable recursive manifold $\mathcal{K}$, locally described by τon field dynamics and globally constrained by Klein curvature invariants. Entanglement arises naturally as topological coupling between locally distinct recursion trajectories sharing global curvature coherence. The unified action integrates the τon Lagrangian, entanglement entropy density, and the Klein curvature term, yielding a recursive field equation that subsumes both informational and physical conservation laws.

1. Introduction: From Shannon to Recursive Geometry

Shannon's entropy defined the measure of uncertainty reduction within a linear, orientable temporal model:

$$H = -\sum_i p_i \log_2 p_i.$$

The UNNS framework replaces probabilistic uncertainty with recursive curvature:

$$H_r = \int \kappa(n)\, d\mu,$$

interpreting information as a topological invariant of transformation rather than a statistical expectation. This reconceptualization introduces the τon—an elementary quantum of recursion curvature—and extends to a full field theory in non-orientable depth space.

2. Recursion as a Geometric Field

Let $\mathcal{N}$ denote the manifold of recursion depths n_i, endowed with curvature κ and torsion τ. The recursive information potential $\Psi_\mu = (\Phi, \vec{\Psi})$ defines the τ-field tensor:

$$\mathcal{T}_{\mu\nu} = \partial_\mu \Psi_\nu - \partial_\nu \Psi_\mu.$$

The τon dynamics obey the field equations:

$$\partial_\nu \mathcal{T}^{\mu\nu} = J_\tau^\mu, \quad \partial_{[\lambda} \mathcal{T}_{\mu\nu]} = 0,$$

with sources $J_\tau^\mu = (\rho_\tau, \vec{J}_\tau)$ describing recursive charge and flux.

3. Klein Manifold and Non-Orientability

The global topology of recursion space is not Euclidean but *Kleinian*. The Klein surface $\mathcal{K}$ satisfies $w_1(\mathcal{K}) \neq 0$, implying a fundamental orientation inversion:

$$S \circ F \circ S = F^{-1}.$$

Thus, local recursion (forward/backward evolution) coexists with global non-orientability—yielding the possibility of *temporal duality* without paradox.

3.1. Klein Curvature Term

We introduce a topological invariant $K_\mathcal{K}$ corresponding to the mean curvature of the non-orientable manifold:

$$K_\mathcal{K} = \int_\mathcal{K} \sqrt{|g|}\, R_\mathcal{K}\, d^4x,$$

where $R_\mathcal{K}$ is the scalar curvature of $\mathcal{K}$. This term enforces global topological coherence in recursive dynamics.

4. Entanglement Entropy as Recursive Coupling

In quantum mechanics, the von Neumann entropy

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho)$$

quantifies the non-separability of subsystems. In the UNNS substrate, entanglement entropy arises from recursive interleaving: two recursion trajectories $a_n^{(A)}$ and $a_n^{(B)}$ share curvature structure across $\mathcal{K}$.

Define the recursive entanglement measure:

$$E_{AB} = \sum_n |\kappa_A(n) - \kappa_B(n)|.$$

When $E_{AB} \to 0$, the trajectories are globally inseparable—entangled through the Klein manifold's curvature continuity.

4.1. Entanglement Density Term

We model local entanglement density $\mathcal{E}_e$ as curvature–torsion coupling:

$$\mathcal{E}_e = \alpha_e \, \vec{\kappa} \cdot \vec{\tau},$$

with α_e the entanglement coupling constant. Integrating over depth gives the total entanglement entropy:

$$S_e = \int \mathcal{E}_e \, d^3x \, dn.$$

This expresses how entanglement corresponds not to probability amplitude overlap, but to recursive curvature coherence.

5. Unified Recursive Field Action

We propose the **Recursive Unified Action**:

$$S_{\text{RU}} = \int_{\mathcal{K}} \left(-\frac{1}{4} \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} + \alpha_e \, \vec{\kappa} \cdot \vec{\tau} + \beta_{\mathcal{K}} \, R_{\mathcal{K}} \right) \sqrt{|g|} \, d^3x \, dn.$$

Here:

- $\mathcal{T}_{\mu\nu}$ — τon field tensor (local recursion flux),
- α_e — entanglement coupling constant,
- $\beta_{\mathcal{K}}$ — Klein curvature coupling parameter,
- $R_{\mathcal{K}}$ — scalar curvature of non-orientable manifold.

5.1. Variation and Field Equations

Variation of S_{RU} with respect to Ψ_μ yields:

$$\partial_\nu \mathcal{T}^{\mu\nu} = J^\mu_\tau + \alpha_e \, \tau^\mu.$$

Variation with respect to the metric $g_{\mu\nu}$ gives the recursive Einstein-type equation:

$$G_{\mu\nu} = \beta_{\mathcal{K}}^{-1} \, T^{(\tau)}_{\mu\nu} + \Lambda_\tau g_{\mu\nu},$$

where

$$T^{(\tau)}_{\mu\nu} = \mathcal{T}_{\mu\alpha} \mathcal{T}_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} \mathcal{T}_{\alpha\beta} \mathcal{T}^{\alpha\beta} + \alpha_e \left(\kappa_\mu \tau_\nu - \frac{1}{2} g_{\mu\nu} \vec{\kappa} \cdot \vec{\tau} \right).$$

The term Λ_τ encodes global recursion curvature—analogous to a cosmological constant in recursive space.

6. Interpretation

6.1. Information–Geometry Equivalence

The three coupled components correspond to:

Local: $\mathcal{T}_{\mu\nu}$ — recursive flux field, Coupling: $\alpha_e\,\vec{\kappa}\cdot\vec{\tau}$ — entanglement density, Global: $\beta_{\mathcal{K}}\,R_{\mathcal{K}}$ — topolog

Together they define the dynamics of information as geometry.

6.2. Conservation of Recursive Entropy

The total recursive entropy density satisfies:

$$\frac{\partial \rho_\tau}{\partial n} + \nabla \cdot \vec{J}_\tau + \frac{\partial \mathcal{E}_e}{\partial n} = 0.$$

This generalizes Shannon's entropy conservation into recursive, entangled manifolds.

6.3. Emergent Physical Analogy

At the macroscopic limit ($R_{\mathcal{K}} \to 0$, $\alpha_e \to 0$), the τon equations reduce to Maxwell's form. At high recursion curvature ($R_{\mathcal{K}} \neq 0$), they produce nonlinear entanglement fields analogous to gravitational–quantum coupling.

7. Philosophical Implications

The Recursive Unified Field reveals that:

- Entropy and curvature are not independent: information loss is geometric folding.
- Entanglement is not nonlocal but nonorientable—arising from the Klein topology of recursion.
- The arrow of time is an emergent projection of a globally non-orientable manifold.

Information, matter, and consciousness may thus be seen as manifestations of a single recursive field, whose curvature, torsion, and coherence generate the apparent structure of reality.

8. Conclusion

The Recursive Field Unification model integrates:

$$\begin{aligned} \text{Local Dynamics:} \quad & \tau\text{-field equations,} \\ \text{Coupling Structure:} \quad & \text{entanglement curvature term,} \\ \text{Global Topology:} \quad & \text{Klein manifold curvature.} \end{aligned}$$

Together they yield a coherent, self-referential geometry of information that extends Shannon entropy into a physical–ontological continuum. This theory suggests that meaning, time, and physical law may all be emergent symmetries of recursive curvature on a non-orientable informational manifold.

"Entropy is curvature; entanglement is its torsion; reality is their recursive unification."