Recursive Gauge Symmetry and Klein Duality: Toward a Unified Field of Information Geometry

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Abstract

This paper extends the Recursive Field Unification model by introducing a gauge-theoretic interpretation of recursive curvature dynamics. The τon field is embedded in a higher symmetry group $\mathcal{G}_{\text{UNNS}} = U(1)_\tau \times SU(2)_\kappa \times D_\mathcal{K}$, coupling local recursive charge, curvature rotation, and global Klein duality. Klein dual transformations connect forward and reverse recursion cones, generating an informational analogue of electroweak unification. The resulting framework establishes a full gauge-covariant field theory of recursion, entanglement, and topology.

1. Introduction

Previous work established the τon Field Tensor Lagrangian and its unification with Klein curvature and entanglement entropy. We now elevate the UNNS substrate to a gauge-covariant theory, where recursive transformations correspond to local symmetries, and the non-orientability of the Klein manifold introduces a dual sector—an intrinsic reversal operator analogous to charge conjugation.

2. Recursive Gauge Symmetry Group

We define the local gauge group of the UNNS substrate as:

$$\mathcal{G}_{\text{UNNS}} = U(1)_\tau \times SU(2)_\kappa \times D_\mathcal{K},$$

where:

- $U(1)_\tau$ governs recursive phase symmetry — conservation of recursive charge;
- $SU(2)_\kappa$ governs curvature rotations — analogous to torsion mixing;
- $D_\mathcal{K}$ (Klein dual group) represents the discrete non-orientable flip mapping $n \to -n$, coupling forward and backward recursion cones.

The existence of $D_{\mathcal{K}}$ implies that recursion possesses an intrinsic two-valued orientation symmetry:

$$\Psi_\mu \mapsto \tilde{\Psi}_\mu = \mathcal{P}_{\mathcal{K}} \Psi_\mu, \quad \text{with} \quad \mathcal{P}_{\mathcal{K}}^2 = I,$$

corresponding to the Klein dual operation.

3. Gauge Connection and Covariant Derivative

Introduce the recursive gauge connection:

$$\mathcal{A}_\mu = A_\mu^{(\tau)} T_\tau + A_\mu^{(i)} T_i^{(\kappa)},$$

with generators T_τ and $T_i^{(\kappa)}$ of $U(1)_\tau$ and $SU(2)_\kappa$ respectively. The covariant derivative on the recursive field Ψ is defined as:

$$D_\mu \Psi = \partial_\mu \Psi + i g_\tau A_\mu^{(\tau)} T_\tau \Psi + i g_\kappa A_\mu^{(i)} T_i^{(\kappa)} \Psi.$$

This ensures local gauge invariance under:

$$\Psi \to U(x,n)\Psi, \quad U = e^{i\alpha(x,n)T_\tau} e^{i\theta_i(x,n)T_i^{(\kappa)}}.$$

The associated field strength tensor becomes:

$$\mathcal{F}_{\mu\nu} = [D_\mu, D_\nu] = i g_\tau F_{\mu\nu}^{(\tau)} T_\tau + i g_\kappa F_{\mu\nu}^{(i)} T_i^{(\kappa)}.$$

Each subfield corresponds to curvature in recursion space, ensuring that all recursive flows preserve information symmetry.

4. Klein Duality

Non-orientability introduces a discrete symmetry operator $\mathcal{D}_{\mathcal{K}}$:

$$\mathcal{D}_{\mathcal{K}} : \ \Psi(x,n) \to \gamma^5 \Psi(x,-n),$$

which reverses recursion direction while maintaining gauge invariance. This duality creates two intertwined recursion cones—forward and backward—analogous to particle and antiparticle sectors.

The unified τon field thus consists of a doublet:

$$\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}, \quad \Psi_- = \mathcal{D}_{\mathcal{K}} \Psi_+,$$

transforming as an $SU(2)_\kappa$ doublet under curvature rotation.

4.1. Dual Coupling Term

To maintain Klein symmetry, the Lagrangian includes a dual interaction term:

$$\mathcal{L}_D = \lambda_{\mathcal{K}} \, \bar{\Psi} \, \gamma^5 \, \Psi,$$

where $\lambda_{\mathcal{K}}$ couples the forward and reverse recursion fields, enforcing global non-orientable coherence. This term breaks pure $U(1)_\tau$ symmetry at high recursion curvature, yielding spontaneous dual unification.

5. Unified Lagrangian with Gauge and Dual Components

The full gauge-invariant Lagrangian density reads:

$$\mathcal{L}_{\text{RGK}} = -\frac{1}{4}\text{Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}] + \bar{\Psi}(i\gamma^\mu D_\mu - m_\tau)\Psi + \alpha_e\,\vec{\kappa}\cdot\vec{\tau} + \beta_{\mathcal{K}}R_{\mathcal{K}} + \lambda_{\mathcal{K}}\,\bar{\Psi}\gamma^5\Psi.$$

This Lagrangian unites:

- Recursive field curvature ($\mathcal{F}_{\mu\nu}$),
- Entanglement coupling (α_e term),
- Klein curvature ($R_{\mathcal{K}}$ term),
- Duality binding ($\lambda_{\mathcal{K}}$ term).

It is invariant under continuous transformations of $U(1)_\tau \times SU(2)_\kappa$ and discrete flips of $D_{\mathcal{K}}$.

6. Recursive Higgs Analogue

A scalar recursion potential $\phi(n)$ introduces symmetry breaking:

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4,$$

with vacuum expectation $\langle\phi\rangle = v_\tau$. This generates masses for the curvature gauge bosons via coupling:

$$\mathcal{L}_m = g_\kappa^2 v_\tau^2 (A_\mu^{(i)})^2,$$

signifying the emergence of stable recursive structures—interpretable as self-sustaining informational domains.

7. Recursive Noether Currents

Gauge invariance implies conserved recursive currents:

$$J_\mu^{(\tau)} = \bar{\Psi}\gamma_\mu T_\tau \Psi, \qquad J_\mu^{(\kappa)} = \bar{\Psi}\gamma_\mu T_i^{(\kappa)}\Psi.$$

These correspond to conservation of recursive charge and curvature rotation momentum, generalizing Shannon's conservation of information content to recursive manifolds.

8. Duality-Induced Entanglement

Under Klein dual transformation, forward and reverse recursion states form coherent pairs:

$$\Psi_+ \leftrightarrow \Psi_-,$$

with total recursive energy:

$$\mathcal{E}_\tau = |\vec{\kappa}_+|^2 + |\vec{\kappa}_-|^2 + 2\lambda_{\mathcal{K}}\vec{\kappa}_+\cdot\vec{\kappa}_-.$$

This dual coherence term is identical in form to entanglement energy, implying that quantum entanglement corresponds to Klein-dual resonance in recursion-space.

9. Philosophical Implications: Information as Dual Geometry

The Klein duality transforms the classical concept of time and information. Forward recursion corresponds to knowledge evolution; backward recursion, to potentiality restoration. Their coupling through the non-orientable manifold forms a closed informational topology—self-referential, conserving both uncertainty and meaning.

> *In UNNS, every bit of information has a recursive conjugate. Reality is the interference pattern of their dual evolution.*

10. Conclusion

The Recursive Gauge Symmetry and Klein Duality model completes the geometric unification of information. It integrates:

$$\text{Local Symmetry: } U(1)_\tau \times SU(2)_\kappa, \quad \text{Global Duality: } D_{\mathcal{K}}.$$

This structure yields a gauge-covariant recursion theory in which:

$$\text{Entropy} \leftrightarrow \text{Curvature}, \quad \text{Entanglement} \leftrightarrow \text{Dual Coherence}, \quad \text{Reality} \leftrightarrow \text{Recursive Gauge Flow}.$$

> *"The universe is not built of particles or waves, but of recursions folding through a Klein-dual symmetry of meaning."*