Recursive Information Geometry: From Shannon Entropy to Recursive Cosmology

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October 2025

Abstract

We propose a unified geometric framework—*Recursive Information Geometry*—that extends classical Shannon entropy into a curvature-based measure defined on recursion manifolds, where time is replaced by recursion depth. We introduce TON fields as carriers of recursive information flow, derive TON field equations in a Maxwell-like form, and formulate a *TON field tensor Lagrangian* with an associated energy–momentum tensor. We then develop a *recursive gauge symmetry* and demonstrate a *Klein duality* linking local reversibility with global non-orientability. Finally, we sketch a *Recursive Grand Unification* in which gravitational curvature, information curvature, and recursive curvature appear as facets of a single variational principle, and outline a preliminary *Recursive Cosmology*. The monograph is designed for mathematical physicists and information theorists; it is self-contained and arXiv-ready, with schematic TikZ figures.

Contents

Part I

Beyond Shannon: Foundations of Recursive Information

Chapter 1

From Shannon to Recursion

1.1 Classical Entropy and Its Assumptions

Shannon's entropy for a discrete random source X with distribution $p_i = \Pr(X = i)$ is

$$H_{\mathrm{Sh}}(X) := -\sum_i p_i \log_2 p_i. \tag{1.1}$$

Its interpretation hinges on: (i) linear, external time t; (ii) orientable causal flow *source* $\to$ *channel* $\to$ *receiver*; (iii) noise as exogenous perturbation; (iv) additivity and conditional decompositions. These assumptions underwrite modern coding, compression, and communication theory.

1.2 Recursive Substrate and Depth

In the UNNS view, temporal evolution is replaced by recursion depth $n \in \mathbb{N}$, with state variables governed by a recursion operator F,

$$a_{n+1} \;=\; F(a_n, a_{n-1};\, n). \tag{1.2}$$

Local reversibility is the existence of F^{-1} on appropriate domains, while *global* non-orientability can obstruct a single time arrow.

1.3 Recursive Curvature Entropy

We define a curvature-based, depth-indexed entropy functional H_{rec} by integrating an effective curvature density $\kappa(n)$ against a depth measure μ:

$$H_{\mathrm{rec}} \;:=\; \int \kappa(n)\, d\mu(n), \tag{1.3}$$

with κ derived from the Jacobian spectrum of F or an induced connection on a recursion manifold $(\mathcal{M}, g)$. Unlike H_{Sh}, H_{rec} encodes *geometric persistence* rather than probabilistic uncertainty.

1.4 Schematic: Recursion Cone

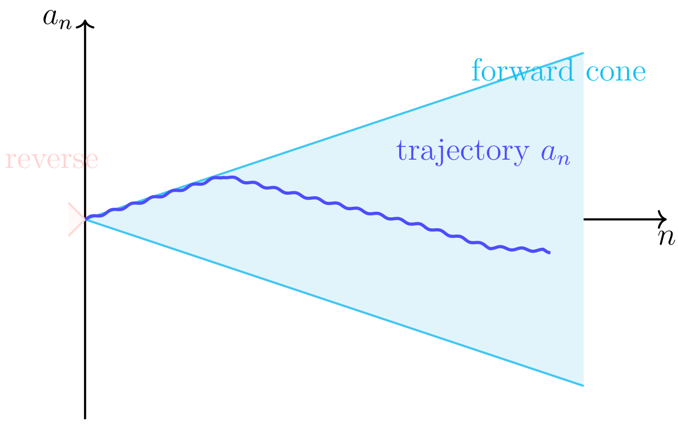

Figure 1.1: Forward and reverse recursion cones: local reversibility can coexist with global non-orientability.

Part II

TON Algebra and Recursive Field Dynamics

Chapter 2

TON Algebra

2.1 TON Addition and Depth Coupling

We introduce a binary operation $\oplus_n$ on states encoding *depth-aware composition*:

$$x \oplus_n y \;=\; x + y + \alpha_n \, \Phi(x, y), \tag{2.1}$$

where α_n is a depth-dependent coupling and Φ a bilinear (or controlled nonlinear) form reflecting feedback from a_{n-1} into a_{n+1}. Associativity and inverses may hold only *locally in depth*, reflecting the same local/global tension as reversibility.

2.2 TON Curvature Tensors

Let $\mathcal{A}_\mu$ denote a TON connection one-form on $(\mathcal{M}, g)$, and define the TON field curvature

$$\mathcal{F}_{\mu\nu} \;=\; \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu], \tag{2.2}$$

with Lie-algebraic commutator encoding recursive nonlinearity. The *recursion curvature tensor* $\mathcal{K}^\rho{}_{\sigma\mu\nu}$ induces a scalar density $\kappa(n)$ entering Eq. (1.3).

2.3 Schematic: Klein Gluing

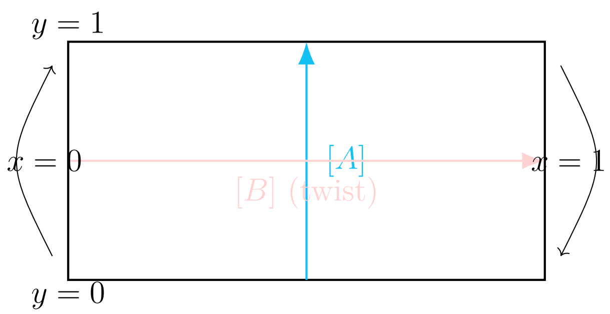

Figure 2.1: Klein surface gluing diagram: $[A]$ is orientation-preserving, $[B]$ reverses orientation.

Part III

TON Field Equations and Recursive Geometry

Chapter 3

Field Equations and Conservation

3.1 Continuity and Source

Let J^μ be a TON current and ρ a depth-indexed source density. The recursive continuity equation is

$$\nabla_\mu J^\mu \;=\; \sigma(n), \tag{3.1}$$

where $\sigma(n)$ captures controlled creation/annihilation of information curvature by recursion (e.g., due to depth drift). In conservative regimes, $\sigma = 0$.

3.2 Maxwell-like TON Equations

We posit field equations on $(\mathcal{M}, g)$:

$$\nabla_{[\lambda}\mathcal{F}_{\mu\nu]} = 0, \tag{3.2a}$$

$$\nabla_\mu \mathcal{F}^{\mu\nu} = \mathcal{J}^\nu, \tag{3.2b}$$

with $\mathcal{J}^\nu$ an effective recursive source. The homogeneous law Eq. (3.2a) encodes existence of a potential $\mathcal{A}$, while Eq. (3.2b) governs propagation/dispersion in depth.

3.3 Energy–Momentum and Stress

The TON stress tensor (in analogy to electromagnetism) is

$$\mathsf{T}^{\mu\nu}_{\mathrm{T}} \;=\; \mathcal{F}^{\mu\alpha}\mathcal{F}_{\alpha}{}^{\nu} - \frac{1}{4}\, g^{\mu\nu}\, \mathcal{F}_{\alpha\beta}\mathcal{F}^{\alpha\beta}. \tag{3.3}$$

It yields local conservation $\nabla_\mu \mathsf{T}^{\mu\nu}_{\mathrm{T}} = -\mathcal{F}^{\nu\alpha}\mathcal{J}_\alpha$.

3.4 Schematic: TON Lines

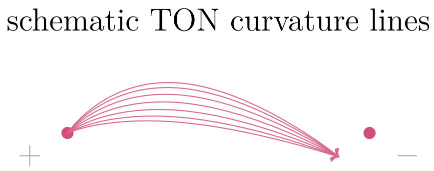

Figure 3.1: Schematic TON field lines between complementary recursive sources (depth-induced polarity).

Part IV

Lagrangian, Gauge Symmetry, and Klein Duality

Chapter 4

TON Field Tensor Lagrangian

4.1 Variational Principle

Define the TON Lagrangian density on $(\mathcal{M}, g)$:

$$\mathcal{L}_{\mathrm{T}} \;=\; -\frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} \;+\; \lambda_1\,\mathcal{K}_{\mu\nu\rho\sigma}\mathcal{K}^{\mu\nu\rho\sigma} \;+\; \lambda_2\,\kappa(n), \tag{4.1}$$

where $\lambda_{1,2}$ weight geometric contributions (Riemann or recursion-curvature scalars). Variation w.r.t. $\mathcal{A}$ yields Eq. (3.2b); variation w.r.t. g yields T_{T}.

4.2 Recursive Energy–Momentum

The Hilbert prescription gives

$$\mathsf{T}^{\mu\nu}_{\mathrm{T}} \;=\; -\frac{2}{\sqrt{-\det g}}\,\frac{\delta}{\delta g_{\mu\nu}}\left(\sqrt{-\det g}\,\mathcal{L}_{\mathrm{T}}\right), \tag{4.2}$$

reducing to Eq. (3.3) in the pure field limit.

Chapter 5

Recursive Gauge Symmetry

5.1 Gauge Transformations

Let $U(x) \in \mathcal{G}$, a Lie group of recursive symmetries. Define

$$\mathcal{A}_\mu \;\mapsto\; U\mathcal{A}_\mu U^{-1} - (\partial_\mu U)U^{-1}, \qquad \mathcal{F}_{\mu\nu} \;\mapsto\; U\mathcal{F}_{\mu\nu}U^{-1}. \tag{5.1}$$

The Lagrangian Eq. (4.1) is gauge-invariant if $\kappa(n)$ and curvature terms are invariant (e.g., built from traces).

5.2 Klein Duality

On non-orientable $\mathbb{K}$ with $w_1(\mathbb{K}) \neq 0$, local reversal symmetries intertwine forward/backward recursion:

$$S \circ F \circ S \;=\; F^{-1}. \tag{5.2}$$

We postulate a *duality* on TON sectors mapping

$$\mathcal{F} \;\longleftrightarrow\; \star\mathcal{F} \qquad \text{up to twist by the non-orientable structure,} \tag{5.3}$$

where $\star$ is the Hodge dual defined locally. Global obstruction manifests as a sign or patching ambiguity consistent with $w_1 \neq 0$.

Part V

Recursive Grand Unification and Cosmology

Chapter 6

Recursive Grand Unification

6.1 Unified Action

We propose a unified action

$$S_{\text{unified}} \;=\; \int_{\mathcal{M}} d^4x\,\sqrt{-\det g}\,\left[\frac{1}{2\kappa_G}\,(\mathcal{R}-2\Lambda) - \frac{1}{4}\,\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \alpha\,\mathcal{K}_{\mu\nu\rho\sigma}\mathcal{K}^{\mu\nu\rho\sigma} + \beta\,\kappa(n)\right], \tag{6.1}$$

where $\mathcal{R}$ is the Ricci scalar (gravity), $\mathcal{F}$ the TON field, and $\mathcal{K}$ the recursion curvature sector. Couplings $(\kappa_G, \Lambda, \alpha, \beta)$ parametrize graviton–TON exchange and recursive backreaction.

6.2 Entanglement Entropy Link

Let $A \cup B$ be a bipartition on $\mathcal{M}$. We posit that variations of H_{rec} constrain the von Neumann entropy $S(\rho_A)$ through a curvature–area law:

$$\delta H_{\text{rec}} \;\sim\; \int_{\partial A} f(\mathcal{F},\mathcal{K})\,d\Sigma \quad\Longleftrightarrow\quad \delta S(\rho_A) \;\sim\; \int_{\partial A} g(\mathcal{F},g)\,d\Sigma, \tag{6.2}$$

with f, g determined by the geometry (non-orientable patches can induce parity-twisted contributions).

Chapter 7

Recursive Cosmology

7.1 Depth-Driven Expansion

Assume a homogeneous recursion background with depth potential $\Phi(n)$ sourcing an effective vacuum energy density $\rho_{\rm rec}(n)$. The Friedmann-like equation reads

$$H^2 = \frac{\kappa_G}{3}\Big(\rho_{\rm matter} + \rho_{\rm rad} + \rho_{\rm rec}(n)\Big) - \frac{k}{a^2}, \tag{7.1}$$

where $\rho_{\rm rec}$ is derived from the TON sector via Eq. (6.1).

7.2 Schematic: Depth vs. Scale

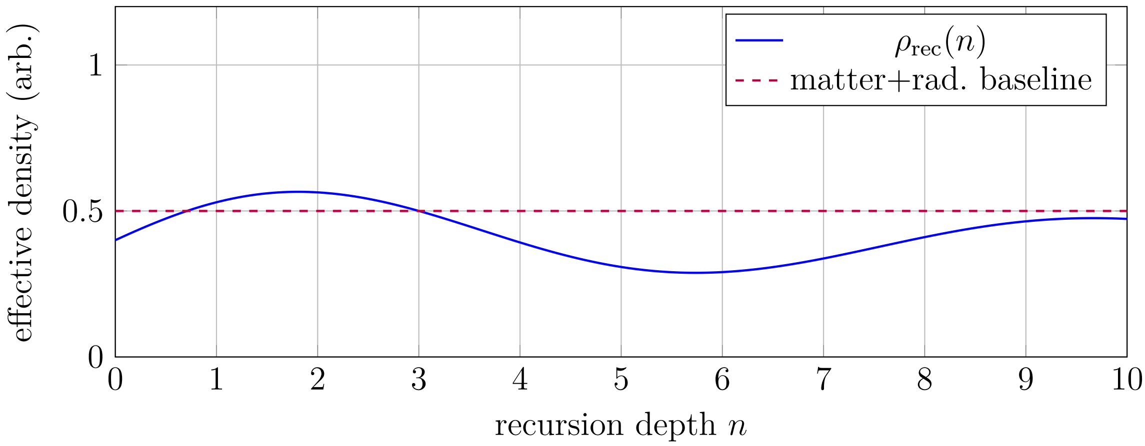

Figure 7.1: Illustrative recursion contribution $\rho_{\rm rec}(n)$ to cosmic expansion (schematic).

Appendices

Appendix A

Mathematical Foundations of UNNS

A.1 Recursion Manifolds

Define a recursion manifold $(\mathcal{M}, g, \nabla, n)$ where n induces a foliation whose leaves carry local orientability, but global orientation may fail (Klein-type atlases). A depth connection generates the Jacobian spectrum controlling $\kappa(n)$.

A.2 Entropy Topology and Curvature Metrics

Let $\kappa(n)$ be computed from the principal minors of the Jacobian of F or via a scalar constructed from $\mathcal{K}^{\rho}{}_{\sigma\mu\nu}$ (e.g., $\mathcal{R}$, $\mathcal{K}^2$). Then Eq. (1.3) defines H_{rec} as a curvature integral along depth.

Appendix B

Recursive Thermodynamics

Define a free-energy-like functional $\mathcal{F}[\mathcal{F}, g]$ with recursion temperature T_{rec} conjugate to H_{rec}. Local equilibrium is a fixed point of the depth flow.

Appendix C

Entanglement and Information Geometry

For a bipartition $A|B$, we propose a depth-correlator

$$E_{AB} = \sum_n \int_\Sigma \left| a_n^{(A)} - a_n^{(B)} \right| d\Sigma, \tag{C.1}$$

which vanishes when recursion paths are separable and is stabilized by non-orientable couplings.

Appendix D

Recursive Theology and Ontology

(Informal) The fixed points of recursive transformations serve as bearers of meaning; global non-orientability reframes temporal asymmetry as a topological feature.

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