UNNS Decomposing, Adopting, and Evaluating: A Higher-Order Triad Linked to the Tetrad and Octad

UNNS Research Notes

September 24, 2025

Abstract

We extend the operational grammar of the Unbounded Nested Number Sequence (UNNS) framework by introducing a new Triad of operators: *Decomposing*, *Adopting*, and *Evaluating*. These complement the existing Tetrad (Inletting, Inlaying, Repair/Normalization, Trans–Sentifying) and integrate with the Octad expansion. We show that decomposition preserves recursive integrity, adoption grafts compatible nests, and evaluation provides admissibility testing. Together they form a cycle of analysis, judgment, and synthesis. We present formal definitions, a theorem on evaluation-driven admissibility, and a diagrammatic overview of the Triad as a feedback loop chained to the broader UNNS operator families.

1 Introduction

The UNNS substrate is structured around recursive operators that act on nested sequences. Earlier work identified a foundational Tetrad and an extended Octad of such operators. We propose here an additional Triad—Decomposing, Adopting, and Evaluating—that captures higher-order structural transformations. This triad embodies analysis (splitting), synthesis (grafting), and judgment (testing), and serves as a bridge between raw recursion and operator grammar.

2 Definitions

Definition 2.1 (Decomposing)**.** *Decomposing is the operator $\mathcal{D}$ that maps a nest $\mathcal{N}$ to a collection $\{\mathcal{N}_i\}$ such that each $\mathcal{N}_i$ is itself a valid UNNS nest with preserved recurrence integrity. Arbitrary cutting is forbidden; decomposition must respect recurrence invariants (echo residues, spectral factors).*

Definition 2.2 (Adopting)**.** *Adopting is the operator $\mathcal{A}$ that embeds a foreign nest $\mathcal{N}_B$ into a host $\mathcal{N}_A$, yielding a grafted nest $\widetilde{\mathcal{N}}$. Adoption requires compatibility of coefficients, depth alignment, or repair adjustment.*

Definition 2.3 (Evaluating)**.** *Evaluating is the operator $\mathcal{E}$ that assigns a stability and resonance profile to a nest $\mathcal{N}$:*

$$\mathcal{E}(\mathcal{N}) = \{\rho(C), \lambda_i, \textit{residue norms}\},$$

where C is the companion matrix and λ_i the echo constants. Evaluation ensures admissibility of adopted or decomposed nests.

3 Formal properties

Lemma 3.1 (Integrity of decomposition)**.** *If $\mathcal{N}$ is a valid nest and $\mathcal{D}(\mathcal{N}) = \{\mathcal{N}_i\}$, then each $\mathcal{N}_i$ satisfies a recurrence rule with coefficients in the same admissible ring as $\mathcal{N}$.*

Proposition 3.2 (Adoption compatibility)**.** *If $\mathcal{A}(\mathcal{N}_A, \mathcal{N}_B) = \widetilde{\mathcal{N}}$ succeeds, then the recurrence coefficients of $\widetilde{\mathcal{N}}$ lie in the closure of the union of coefficient rings of $\mathcal{N}_A$ and $\mathcal{N}_B$.*

Theorem 3.3 (Evaluation ensures admissibility)**.** *Suppose $\mathcal{A}(\mathcal{N}_A, \mathcal{N}_B) = \widetilde{\mathcal{N}}$. If $\mathcal{E}(\widetilde{\mathcal{N}})$ satisfies*

$$\rho(C_{\widetilde{\mathcal{N}}}) < 1 \quad \textit{and} \quad \textit{residue norms} < \tau,$$

for a stability threshold τ, then $\widetilde{\mathcal{N}}$ is admissible as a UNNS nest and remains stable under projection and repair.

Proof. Evaluation checks both spectral radius and residue norms. If $\rho(C) < 1$, iteration contracts. If residues lie below threshold τ, repair operators need not trigger, so the nest evolves consistently. Hence admissibility follows. □

4 Integration with Tetrad and Octad

4.1 Tetrad

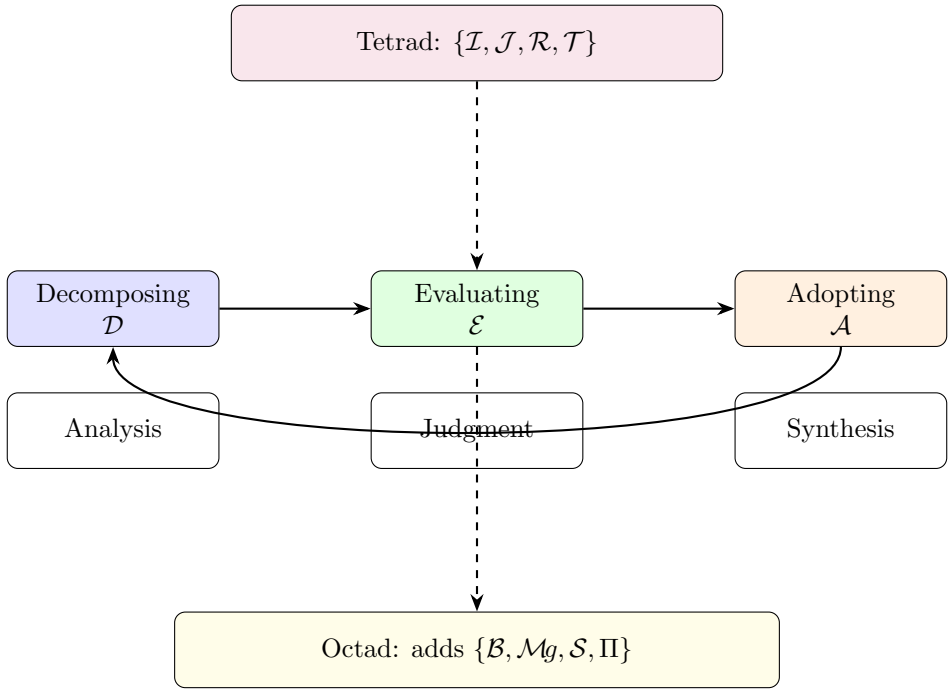
The Tetrad $\{\mathcal{I}, \mathcal{J}, \mathcal{R}, \mathcal{T}\}$ introduces seeding (Inletting, Inlaying), stabilization (Repair), and perceptual mapping (Trans–Sentifying). The Triad $\{\mathcal{D}, \mathcal{A}, \mathcal{E}\}$ builds on this by restructuring nests.

4.2 Octad

The Octad adds Branching, Merging, Shadowing, Projection. Together with the Triad, the operator family expands to eleven members. These can be grouped as:

- Seed and Growth: Inletting, Inlaying, Branching.
- Stability: Repair, Merging, Projection.
- Perception and Masking: Trans–Sentifying, Shadowing.
- Higher-order Structuring: Decomposing, Adopting, Evaluating.

5 Visualization



6 Conclusion

Decomposing, Adopting, and Evaluating extend the UNNS operational grammar beyond the Tetrad and Octad. They provide a higher-order cycle of analysis, judgment, and synthesis that governs the restructuring of nests. By chaining the Triad to the earlier operator families, the grammar achieves a new level of expressive completeness, suitable for applications in logic, physics, and recursive system modeling.