Zero as Nest and Modulus in UNNS

UNNS Research Notes

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Abstract

In the UNNS substrate, zero is more than a trivial value. It functions structurally as a *universal absorbing nest* and algebraically as a *universal modulus anchor*. This expansion develops precise definitions, proves a theorem formalizing these dual roles, and provides a TikZ diagram visualizing zero's behavior in both the nesting and modulus perspectives.

1 Zero as a Nest

Definition 1.1 (Nest)**.** *A* nest $\mathcal{N}$ *is a recursively defined subsequence structure generated by a recurrence relation of order* r*:*

$$u_{n+r} \;=\; \sum_{j=1}^{r} c_j \, u_{n+r-j}, \qquad n \geq 0,$$

with initial data $(u_0, \ldots, u_{r-1})$*.*

Definition 1.2 (Zero Nest)**.** *The* zero nest $\mathcal{N}_0$ *is defined by*

$$\mathcal{N}_0 = \{u_n = 0 \mid n \in \mathbb{Z}_{\geq 0}\}.$$

Remark 1.3. *The zero nest is* absorbing*: once all terms equal zero, further recurrence steps preserve zero. Thus, it is the minimal valid nest structure.*

2 Zero as a Modulus

Definition 2.1 (Modulus congruence)**.** *Given* $m \in \mathbb{N}$*, two sequences* $\{u_n\}, \{v_n\}$ *are congruent mod* m *if*

$$u_n \equiv v_n \pmod{m}, \qquad \forall n.$$

Definition 2.2 (Zero modulus class)**.** *The* zero class *modulo* m *is*

$$[0]_m = \{\, k \in \mathbb{Z} \mid k \equiv 0 \pmod{m} \,\}.$$

Remark 2.3. *For every modulus* m*, the class* $[0]_m$ *is universal: it is the additive identity class, and every class reduces to it by multiplication with* m*.*

3 Main Theorem: Dual Role of Zero

Theorem 3.1 (Zero as universal nest and modulus anchor)**.** *In the UNNS substrate:*

1. *(Nest property)* $\mathcal{N}_0$ *is a universal absorbing nest: for any recurrence of finite order with coefficients* $\{c_j\}$*, the zero nest satisfies the recurrence and absorbs collapse mappings.*

2. *(Modulus property) For any modulus* $m \geq 2$*, the zero class* $[0]_m$ *is the universal modulus anchor: every recurrence sequence* $\{u_n\}$ *satisfies*

$$u_n \equiv 0 \pmod{m} \quad \textit{iff } u_n \in [0]_m,$$

and $[0]_m$ *is the only class preserved under multiplication by* m*.*

Proof. (1) Substituting $u_n = 0$ for all n into any recurrence yields

$$u_{n+r} = \sum_{j=1}^{r} c_j \cdot 0 = 0,$$

so $\mathcal{N}_0$ satisfies every recurrence. If a nest collapses to all zeros, it remains in $\mathcal{N}_0$, so the nest is absorbing.

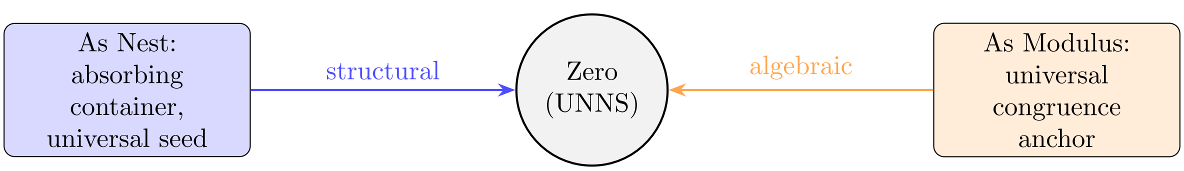
(2) By definition, $[0]_m$ is closed under addition and multiplication. For any $k \in \mathbb{Z}$, $m \cdot k \equiv 0 \pmod{m}$, so multiplication by m always maps into $[0]_m$. No other congruence class has this property. Thus $[0]_m$ is the universal modulus anchor. □

4 Examples

Example 4.1 (Fibonacci seed)**.** *The Fibonacci sequence* $(0, 1, 1, 2, 3, 5, \ldots)$ *uses* $u_0 = 0$ *as part of its initial nest. Here, zero is the seed nest placeholder anchoring the lattice.*

Example 4.2 (Modulus collapse)**.** *Consider* $u_{n+1} = 2u_n$ *with* $u_0 = 3$*. Mod* 6*, we have* $3, 0, 0, 0, \ldots$*, collapsing immediately into* $[0]_6$*. Thus zero acts as the modulus anchor.*

5 Diagram: Dual Role of Zero



6 Conclusion

Zero in UNNS is not trivial. As a *nest*, it is the universal absorbing container that recurrences can collapse into or restart from. As a *modulus*, it is the universal congruence anchor against which invariants and residues are measured. This duality makes zero both a structural and an algebraic cornerstone of the UNNS discipline.