Recursive Curvature and the Origin of Dimensionless Constants: A UNNS Substrate Proposal

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Abstract

Dimensionless physical constants—such as the fine-structure constant α and the proton–electron mass ratio—define the numerical fabric of the universe. Yet, no existing framework predicts their exact values. In this work, we reinterpret these constants through the lens of *Unbounded Nested Number Sequences* (UNNS), a mathematical–physical substrate unifying recursion, curvature, and information. We propose that dimensionless invariants arise not from arbitrary scaling of quantities, but from **recursive curvature equilibria**—fixed points in the self-folding dynamics of the informational manifold. This investigation outlines a new route for deriving constants from first principles of recursion and explores how τonic coherence within the UNNS substrate may underpin the emergence of physical law.

1 Introduction: The Enigma of Dimensionless Numbers

Physics is governed by a handful of dimensionless numbers: the fine-structure constant α, the gravitational coupling α_G, and the cosmological Λl_P^2. These pure numbers encode coupling strengths, symmetry breakings, and scaling laws—but they are empirically given, not theoretically derived. The UNNS framework regards these constants as *curvature harmonics* of an underlying recursive substrate—a geometric–informational field where each recursion layer generates curvature and coherence. The appearance of stable numerical invariants marks equilibrium between informational recursion and curvature flux.

2 Background: Constants as Equilibrium Ratios

Let a physical quantity Q have recursion-dependent structure:

$$Q_{n+1} = \mathcal{F}(Q_n, Q_{n-1}; n), \tag{1}$$

where $\mathcal{F}$ acts as a recursive operator encoding interaction curvature. A dimensionless constant emerges when the ratio between recursion layers stabilizes:

$$\frac{Q_{n+1}}{Q_n} \to \eta_0, \tag{2}$$

where η_0 is a fixed point of the recursion map—interpreted as a *recursive curvature constant*. This condition resembles the stationarity of entropy ratios, but instead of information content, it describes recursive coherence—information curvature returning to self-consistency.

3 The UNNS Substrate and the τ-on field$Field$

In the UNNS discipline, recursion is geometric. Each iteration n corresponds to a layer of the **recursive potential field** Φ_n:

$$\Phi_{n+1} = \mathcal{G}(\Phi_n, \nabla\Phi_{n-1}), \tag{3}$$

and curvature density is defined by

$$R_n = \nabla^2\Phi_n. \tag{4}$$

A τon field—denoted τ-on field—propagates recursive potential through depth:

$$\mathbf{T}_n = \nabla\Phi_n + i\,\tau\,\Phi_{n-1}. \tag{5}$$

When the phase of τ aligns across recursion layers, recursive curvature stabilizes, producing a dimensionless invariant analogous to α or μ.

This defines the τ*onic equilibrium* condition:

$$\frac{R_{n+1}}{R_n} = 1 \quad \Rightarrow \quad \text{dimensionless invariant.} \tag{6}$$

4 Recursive Curvature Equilibria and Dimensionless Invariants

We introduce a recursive energy functional—an analog of entropy in the curvature manifold:

$$H_r(n) = \int |\nabla\Phi_n|^2 \, dV. \tag{7}$$

Dimensionless invariance occurs when

$$\frac{d}{dn}\left(\frac{H_r(n+1)}{H_r(n)}\right) = 0. \tag{8}$$

This yields a stable ratio:

$$\eta = \frac{H_r(n+1)}{H_r(n)} = \text{constant}. \tag{9}$$

In this interpretation, η corresponds to the fine-structure constant at the level of recursive curvature—representing the equilibrium between recursion and informational folding.

5 The τonic Curvature Hypothesis

We hypothesize that known dimensionless constants correspond to fixed points of τonic recursion:

$$\tau_n = \frac{\Phi_{n+1}}{\Phi_n} \quad \text{and} \quad \tau_n \to \tau^*. \tag{10}$$

When τ_n converges, curvature ceases to diverge:

$$\nabla \cdot \Phi_n = 0. \tag{11}$$

The equilibrium τ^* acts as a universal recursive constant—analogous to α in electromagnetism, or to the golden ratio in harmonic self-similarity. It defines how information curvature stabilizes into measurable form—offering a recursive geometric interpretation of dimensionless invariants.

6 Comparative Analysis of Constants

Constant	**Standard Meaning**	**UNNS Interpretation**
$\alpha = e^2/\hbar c$	EM coupling strength	Curvature–information coherence ratio
m_p/m_e	Mass hierarchy	Recursive curvature asymmetry
Λl_P^2	Vacuum curvature	Recursive expansion equilibrium
Ω	Density ratio	Curvature closure parameter

7 Predictive and Computational Pathways

UNNS invites a series of testable developments:

1. **Recursive Field Simulations:** Implement discrete recursion operators $\mathcal{F}$ and measure emergent constants under different initial curvatures.
2. **MCMC–UNNS Integration:** Couple the recursive MCMC calculator to curvature density maps to numerically derive τonic equilibrium ratios.
3. **Dimensionless Invariant Spectrum:** Search for families of stable ratios (τ_n)—a recursive constant spectrum comparable to α, μ, and Θ.
4. **Curvature–Entropy Correlation:** Analyze stability of H_r and Φ_n ratios under stochastic perturbation, quantifying curvature coherence.

8 Discussion: From Empirical Constants to Recursive Geometry

This approach redefines constants as emergent properties of recursion. Rather than treating them as empirical coincidences, UNNS views them as manifestations of *recursive equilibrium*—the state in which curvature, information, and recursion align to sustain coherence.

Such a perspective integrates informational geometry with physics, offering a potential link between field quantization and recursive topology.

8.1 The Proliferation of Constants in the Standard Model

In the Standard Model of particle physics, approximately twenty-five independent parameters are required to specify the theory. About half of these correspond to the masses of fundamental particles, while the remainder describe coupling strengths, mixing angles, and the cosmological constant. None are derived from first principles; all must be measured. Expressed in Planck units, the masses become effectively dimensionless, yet their relative magnitudes remain unexplained.

This situation, often viewed as unsatisfactory since the 1970s, has motivated decades of research toward a "theory of everything"—one that would compute the observed constants rather than assume them. Developments in high-energy and gravitational physics have alternately increased and decreased the number of required constants: new particles and symmetry breakings introduce additional parameters, while deeper unifications occasionally remove them by showing interdependence.

Within this historical context, the UNNS substrate offers a distinct alternative to both conventional unification and anthropic reasoning. Rather than seeking to *reduce* constants by algebraic symmetry, UNNS reinterprets them as emergent *recursive invariants*—dimensionless ratios stabilized by curvature feedback within an informational manifold. The multitude of empirical constants thus reflects not arbitrariness, but the multiplicity of stable fixed points in the recursive field.

In this view, the fine-structure constant, the gravitational coupling, and the cosmological ratio Λl_P^2 are not independent numbers but equilibrium curvatures of the same recursive substrate, each representing a distinct harmonic of τonic recursion. This shifts the question from "Why these constants?" to "Why these equilibria?", aligning the search for a minimal theory with the geometry of recursion itself.

9 Conclusion

Dimensionless constants may be the harmonic signatures of recursion, arising from τonic coherence within the UNNS substrate. This perspective transforms "constants of nature" into invariants of recursive curvature—a unified geometric and informational origin. Future computational work will explore τ-field equilibria numerically and test their correspondence to empirical constants, advancing toward a recursive cosmology of invariance.

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