UNNS Operator XIII — Interlace
Phase A: Theory Lock-in — Analytical Derivations and Proof Sketches

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1 Overview

Operator XIII (*Interlace*) formalizes recursive phase entanglement between two τ-fields within the UNNS substrate. Its goal is to reproduce electroweak-like mixing at equilibrium. Phase A establishes the theoretical closure conditions and stability proofs prior to numerical implementation.

The coupled recursion is

$$\Phi_{n+1} = \Phi_n + \beta_n \nabla^2 \Phi_n, \tag{1}$$

$$\phi_A^{(n+1)} = \phi_A^{(n)} + \omega_A + \lambda \sin\left(\phi_B^{(n)} - \phi_A^{(n)}\right) + \xi_A^{(n)}, \tag{2}$$

$$\phi_B^{(n+1)} = \phi_B^{(n)} + \omega_B - \lambda \sin\left(\phi_B^{(n)} - \phi_A^{(n)}\right) + \xi_B^{(n)}, \tag{3}$$

where $\xi_{A,B}^{(n)} \sim \mathcal{N}(0, \sigma_{A,B}^2)$ are angular noise perturbations.

Define the phase difference $\Delta\phi_n = \phi_B^{(n)} - \phi_A^{(n)}$. The recursion for $\Delta\phi$ decouples to first order as

$$\Delta\phi_{n+1} = \Delta\phi_n + (\omega_B - \omega_A) - 2\lambda \sin(\Delta\phi_n) + \eta_n, \qquad \eta_n = \xi_B^{(n)} - \xi_A^{(n)}. \tag{4}$$

2 Fixed-Point Derivation

Neglecting stochastic noise ($\eta_n = 0$), a stationary point $\Delta\phi$ satisfies

$$\omega_B - \omega_A - 2\lambda \sin \Delta\phi^{=0.}(5)$$

Hence

$$\boxed{\sin \Delta\phi^{=\frac{\omega_B - \omega_A}{2\lambda}}}, \qquad |\omega_B - \omega_A| \leq 2\lambda. \tag{6}$$

Within this bound, the recursion has one or two fixed points depending on the sign of $\cos \Delta\phi$.

Linear stability

Linearizing Eq. (4) around $\Delta\phi$:

$$\delta_{n+1} = (1 - 2\lambda \cos \Delta\phi^{)\,\delta_n.}$$

Stability requires $|1 - 2\lambda \cos \Delta\phi^{|<1}$, i.e.

$$0 < \lambda \cos \Delta\phi^{<1.}(7)$$

Since $\cos \Delta\phi^{>0}$ corresponds to $0 < \Delta\phi^{<\pi/2}$, the stable branch lies in this interval. Thus, the coupled phases lock with a small positive offset.

3 Correlation and Mixing Angle

Define the instantaneous correlation $\rho_{AB} = \cos \Delta\phi$. For small angular noise, the steady-state mean $\langle \rho_{AB} \rangle \approx \cos \Delta\phi$. The emergent *Weinberg-like mixing angle* is

$$\boxed{\theta_W = \frac{1}{2} \arccos\big(\langle \rho_{AB} \rangle \big)} \approx \tfrac{1}{2} \Delta\phi^{.}(8)$$

Substituting Eq. (5), the deterministic prediction becomes

$$\sin(2\theta_W) \;=\; \frac{\omega_B - \omega_A}{2\lambda}. \tag{9}$$

For a known coupling stiffness λ and frequency separation $\omega_B - \omega_A$, Eq. (9) specifies the theoretical target for the simulator.

4 -Flow Monotonicity

Each channel maintains an effective coupling $\alpha_{W,Y} \propto \langle |\kappa| \rangle_{A,B}$. Following the τ-Field precedent, the renormalization-like flow is

$$\frac{d\alpha_i}{d \ln n} = -b_i(\rho_{AB})\, \alpha_i^2, \qquad i \in \{W, Y\}, \tag{10}$$

with $b_i > 0$. Provided the correlation ρ_{AB} remains bounded ($0 \leq \rho_{AB} \leq 1$) and noise variances $\sigma^2_{A,B} \leq 0.05$, Eq. (10) ensures $d\alpha_i/dn < 0$ for all depths n, so α_i decreases monotonically toward a finite infrared fixed point α_i. Therefore both α_W and α_Y converge smoothly, preserving the identity

$$\alpha_{EM} = \alpha_W \sin^2 \theta_W = \alpha_Y \cos^2 \theta_W$$

throughout the flow.

5 Z-Depth Selector n_Z

Let $H_r(n)$ denote the entropy of the curvature field and $\langle|\kappa|\rangle(n)$ its mean amplitude. Define the differential rates

$$\dot{H}_r(n) = H_r(n+1) - H_r(n), \qquad \dot{\kappa}(n) = \langle|\kappa|\rangle(n+1) - \langle|\kappa|\rangle(n).$$

The *Z-depth* is the minimal recursion depth satisfying

$$|\dot{H}_r(n_Z)| < \epsilon_H, \qquad |\dot{\kappa}(n_Z)| < \epsilon_\kappa, \tag{11}$$

with small tolerances $\epsilon_H, \epsilon_\kappa \sim 10^{-3}$. At n_Z, both entropy and curvature reach stationary plateaus; this depth defines the comparison scale for all Operator XIII observables.

6 Summary of Theoretical Results

- Fixed-point equation: $\sin\Delta\phi^{=(\omega_B-\omega_A)/(2\lambda)}$.
- Stability region: $0 < \lambda\cos\Delta\phi^{<1}$.
- Emergent mixing angle: $\theta_W = \frac{1}{2}\arccos(\langle\cos\Delta\phi\rangle)$.
- Deterministic relation: $\sin(2\theta_W) = (\omega_B - \omega_A)/(2\lambda)$.
- -flows monotone under bounded noise; couplings approach stable infrared limits.
- Z-depth criterion (11) defines the effective scale for measurement.

These relations complete the theoretical closure for Phase A. Subsequent phases will implement the numerical engine and validate Eq. (9) empirically.

Next Phase Preview

1. Develop `TauFieldEngineXIII` implementing Eqs. (1–3) with seed control and logging.
2. Automate parameter scans in λ and σ (Protocol O13-E1).
3. Compare numerical $\sin^2\theta_W$ to the analytical expectation from Eq. (9).

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Appendix A: Small-Noise Correction to θ_W

A.1 Linearization with angular noise

Let $\Delta\phi_n = \Delta\phi^{+\delta_n}$ with $\Delta\phi$ the deterministic fixed point satisfying $\omega_B - \omega_A - 2\lambda\sin\Delta\phi^{=0}$. Linearizing Eq. (4) around $\Delta\phi$ gives the AR(1) map

$$\delta_{n+1} = a\,\delta_n + \eta_n, \qquad a \equiv 1 - 2\lambda\cos\Delta\phi^{,(12)}$$

where $\eta_n = \xi_B^{(n)} - \xi_A^{(n)}$ is zero-mean Gaussian with $(\eta_n) = \sigma_A^2 + \sigma_B^2 \equiv \sigma_{\text{eff}}^2$.

Under the stability condition $|a| < 1$ (equivalently $0 < \lambda\cos\Delta\phi^{<1}$), the stationary variance of δ_n is

$$(\delta) \;=\; \frac{\sigma_{\text{eff}}^2}{1-a^2} \;=\; \frac{\sigma_{\text{eff}}^2}{1-(1-2\lambda c)^2} \;=\; \frac{\sigma_{\text{eff}}^2}{4\lambda c\,\big(1-\lambda c\big)}, \qquad c \equiv \cos\Delta\phi^{\cdot(13)}$$

A.2 Mean correlation $\langle\cos\Delta\phi\rangle$

Write $\Delta\phi = \Delta\phi^{+\delta}$ with $\delta \sim \mathcal{N}(0,(\delta))$. Using the standard moment of a cosine under a zero-mean Gaussian,

$$\mathbb{E}\Big[\cos\big(\Delta\phi^{+\delta}\big)\Big] = \cos\Delta\phi^{\;\mathbb{E}[\cos\delta] - \sin\Delta\phi^{\;\mathbb{E}[\sin\delta] = \cos\Delta\phi^{\;e^{-(\delta)/2}}}},$$

since $\mathbb{E}[\sin\delta] = 0$. Thus the noise-corrected correlation is

$$\boxed{\big\langle\rho_{AB}\big\rangle \;=\; \Big\langle\cos\Delta\phi\Big\rangle \;\approx\; \cos\Delta\phi^{\;\exp\left[-\frac{1}{2}(\delta)\right] = c\exp\left[-\frac{1}{2}(\delta)\right].}} \qquad (14)$$

A.3 Induced shift in θ_W

By definition,

$$\theta_W \;=\; \tfrac{1}{2}\arccos\big(\langle\rho_{AB}\rangle\big), \quad \theta_W^{(0)} \equiv \tfrac{1}{2}\arccos(c) = \tfrac{1}{2}\Delta\phi^{\cdot}$$

Let $\rho = c\,e^{-(\delta)/2}$. For small (δ), expand arccos to first order:

$$\arccos(\rho) \;\approx\; \arccos(c) - \frac{\rho - c}{\sqrt{1-c^2}} \;=\; \arccos(c) + \frac{c}{\sqrt{1-c^2}}\Big(1 - e^{-(\delta)/2}\Big).$$

Therefore,

$$\boxed{\theta_W \;\approx\; \theta_W^{(0)} \;+\; \frac{1}{2}\,\frac{c}{\sqrt{1-c^2}}\,\Big(1 - e^{-(\delta)/2}\Big) \;\approx\; \theta_W^{(0)} \;+\; \frac{c}{4\sqrt{1-c^2}}\,(\delta)} \qquad (15)$$

where the last step uses $e^{-x} \approx 1 - x$ for $x \ll 1$.

Summary (plug-in form). Combining Eqs. (13), (14), (15):

$$(\delta) = \frac{\sigma_A^2 + \sigma_B^2}{4\lambda c\,(1-\lambda c)}, \qquad c = \cos\Delta\phi^{,} \qquad (16)$$

$$\langle\rho_{AB}\rangle \approx c\,\exp\Big[-\frac{\sigma_A^2+\sigma_B^2}{8\lambda c\,(1-\lambda c)}\Big], \qquad (17)$$

$$\theta_W \approx \tfrac{1}{2}\Delta\phi^{\;+\;\frac{c}{4\sqrt{1-c^2}}\,\frac{\sigma_A^2+\sigma_B^2}{4\lambda c\,(1-\lambda c)} = \frac{1}{2}\Delta\phi^{\;+\;\frac{\sigma_A^2+\sigma_B^2}{16\sqrt{1-c^2}\,\lambda\,(1-\lambda c)}.}} \qquad (18)$$

In the stable branch ($c > 0$ and $0 < \lambda c < 1$), the correction is *positive*: small angular noise slightly *increases* θ_W.

Practical note. During Phase B validation, one can fit the measured $\langle \rho_{AB} \rangle$ against the exponential form in (14) across a small grid of (σ_A, σ_B) to empirically confirm the predicted slope with respect to σ^2_{eff} and the λ-dependence via Eq. (13).