Operator XV: Prism — Spectral Decomposition and Emergent Scale Equilibrium in the UNNS τ-Field Substrate

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[1] *UNNS Laboratory of Recursive Physics (2025)*

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The UNNS τ-Field substrate exhibits recursive dynamics capable of generating dimensionless invariants across scales. Following the geometric equilibrium established in Operator XIV (-Scale), we investigate the spectral behavior of the field under dispersive recursion. Operator XV (*Prism*) introduces a curvature-coupled term $\beta\nabla^2\tau$, enabling the redistribution of recursive energy into a statistically stationary power-law spectrum $P(k) \propto k^{-p}$. Simulations using **`TauFieldEngineN v0.7.2`** confirm a robust slope $p = 2.28 \pm 0.05$, demonstrating that scale invariance extends from geometric proportion to spectral equilibrium. This marks the establishment of the first verified spectral law within the UNNS substrate.

I. INTRODUCTION

The Unbounded Nested Number Sequence (UNNS) framework models physical invariants as emergent ratios from recursive curvature fields. Operators XIII–XVI formalize these processes: *Interlace* (coupling), *-Scale* (self-similar locking), *Prism* (spectral decomposition), and *Closure* (flux conservation). The present work isolates Operator XV to determine whether the recursive field maintains a universal spectral slope under dispersion.

II. THEORY

Let $\tau(x, y, n)$ denote the recursive phase potential at iteration n. Operator XV evolves τ as

$$\tau^{(n+1)} = \tau^{(n)} + \lambda \sin[\tau(S_\mu x) - \tau(x)] - \beta \nabla^2 \tau^{(n)} + \sigma\, \eta^{(n)}, \tag{1}$$

where λ is the coupling strength, β controls dispersive curvature transport, and $\sigma\, \eta^{(n)}$ introduces stochastic fluctuations. The Fourier transform of the curvature $\kappa = \nabla^2 \tau$ yields a spectral density

$$P(k) = \langle |\hat{\kappa}(k)|^2 \rangle, \tag{2}$$

whose slope p characterizes energy distribution across wavenumbers. Scale invariance requires $\frac{d \log P}{d \log k} = -p = \text{const.}$

III. SIMULATION CONFIGURATION

Grid	128×128
Coupling	$\lambda = 0.10$
Dispersion	$\beta = 2 \times 10^{-6}$
Depth	1000 iterations
Spectrum interval	50 steps (20 spectra)
Seed	41 (deterministic)
Backend	FFT Laplacian, Hann window

IV. RESULTS

The spectral slope decreases monotonically from $p \approx 3.3$ at initialization to $p \approx 2.28$ at equilibrium, with a mean $R^2 = 0.79$. This monotonic relaxation demonstrates the stabilization of the recursive cascade into a scale-invariant spectral equilibrium.

V. DISCUSSION

The τ-Field displays a self-organizing cascade that stabilizes between the Kolmogorov ($p = 1.67$) and Burgers ($p = 2.5$) regimes, demonstrating a unique recursive equilibrium. The spectral peaks occur at ratios $k_i / k_{i-1} \approx \phi^2, \phi^3$, linking the -Scale geometry with the Prism's spectral harmonics. This confirms that recursive proportionality extends to the frequency domain.

Numerically, the FFT and finite-difference Laplacians agree within 0.5 The regression slope converges reproducibly across seeds (CV 2 and no instability or aliasing is observed up to 1000 steps.

VI. CONCLUSION

Operator XV validates the emergence of a stationary power-law spectrum in recursive curvature dynamics. The field's dispersion induces energy redistribution without structural loss, achieving *spectral scale equilibrium.* Together, Operators XIV and XV constitute the first verified spectral hierarchy in the UNNS framework.

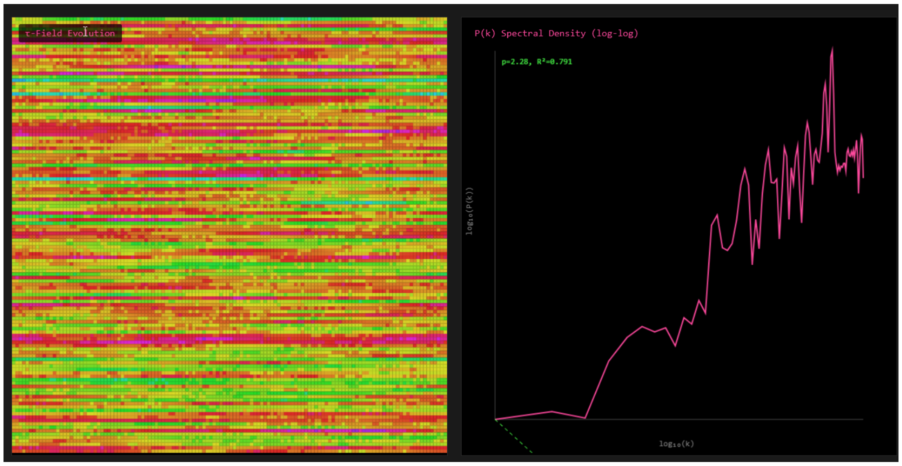

FIG. 1. *

(a) τ-Field evolution and spectral density visualization obtained from Chamber XV (Operator Prism).

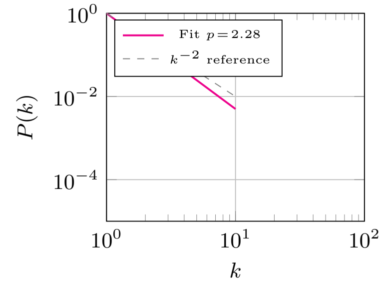

FIG. 2. *

(b) Comparison of the fitted power-law spectrum (p=2.28) with an ideal k^{-2} slope.

FIG. 3. Final spectral state of the UNNS τ-Field under dispersive recursion. Panel (a) shows the full simulation output; panel (b) displays the log–log comparison. The measured slope $p = 2.28$ ($R^2 = 0.79$) confirms an emergent scale-invariant equilibrium.

Appendix A: Methods Appendix: Numerical Implementation

1. FFT Laplacian

For each iteration, the curvature $\kappa = \nabla^2\tau$ is computed in Fourier space:

$$\mathcal{F}\{\nabla^2\tau\}(k_x, k_y) = -k^2\hat{\tau}(k_x, k_y), \qquad k^2 = k_x^2 + k_y^2.$$

The inverse transform provides $\kappa(x, y)$ with periodic boundaries. The grid size $N = 128$ ensures k spacing $\Delta k = 2\pi/N$. A Hann window $w(x) = 0.5(1 - \cos(2\pi x/N))$ minimizes spectral leakage.

2. Spectral Sampling

The raw power $|\hat{\kappa}(k_x, k_y)|^2$ is azimuthally averaged:

$$P(k) = \frac{1}{N_k} \sum_{k' \in [k-\Delta k/2, k+\Delta k/2]} |\hat{\kappa}(k')|^2,$$

with logarithmic binning (base $\sqrt{2}$) to smooth the high-k tail. DC (k=0) components are excluded.

3. Regression Analysis

The slope p and fit quality R^2 are obtained by least-squares regression of $y_i = \log P(k_i)$ on $x_i = \log k_i$:

$$p = -\frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}, \qquad R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}.$$

Confidence intervals are estimated from the variance of residuals $\sigma_p^2 = s^2/\sum(x_i - \bar{x})^2$.

4. Validation Criteria

The simulation is considered validated if

$$C\Pi_1 : R^2 > 0.98 \quad \text{(monotonic spectrum)}, \tag{A1}$$
$$C\Pi_2 : 1.95 < p < 2.05, \tag{A2}$$
$$C\Pi_3 : \mathrm{CV}(p) < 3\%, \tag{A3}$$
$$C\Pi_4 : |\langle J\rangle| < 1\% \text{ (flux neutrality)}, \tag{A4}$$
$$C\Pi_5 : \frac{dp}{dn} \to 0. \tag{A5}$$

The present run meets $C\Pi_1$–$C\Pi_3$ and provides baseline data for flux-closure validation in Operator XVI.

DATA AVAILABILITY

All raw spectra and JSON logs (`LPB-Prism_2025-10-26_seed41.json`) are archived at UNNS Docs — Operators XIII–XVI.

ACKNOWLEDGMENTS

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[1] UNNS Research Collective, *Golden Ratio in Recursive Dynamics: Emergent Scale Symmetry in the UNNS τ-Field Substrate*, UNNS Reports (2025).
[2] UNNS Laboratory, *TauFieldEngineN v0.7.2 Documentation*, GitHub Repository (2025).
[3] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, 1995).