Scale Invariance in Coupled Field Systems: Recursive Coupling and Spectral Equilibrium in the UNNS Substrate

UNNS Research Collective[1]

[1] *UNNS Laboratory of Recursive Physics, 2025*

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We extend the UNNS τ-Field framework to coupled recursive systems, demonstrating that scale invariance persists under multi-field interaction. Introducing inter-field coupling coefficients λ_{ij} and a dispersive term $\beta\nabla^2\tau_i$, we show analytically and numerically that the ensemble spectrum $P_i(k) \propto k^{-p}$ converges toward a universal exponent $p \simeq 2$, independent of configuration. The result reveals scale symmetry as an emergent invariant of coupled curvature dynamics, forming the theoretical basis for Operators XV (Prism) and XVI (Closure) of the UNNS substrate.

I. INTRODUCTION

Scale invariance—self-similarity under magnification—pervades nonlinear physics. Within the UNNS substrate, Operator XIV (-Scale) established the single-field golden-ratio attractor. We now generalize to coupled τ-Fields with recursive cross-interaction, probing whether the invariant scaling law persists in multi-field domains.

II. COUPLED RECURSIVE FORMULATION

For N interacting fields $\{\tau_i\}$,

$$\tau_i^{(n+1)} = \tau_i^{(n)} + \sum_{j=1}^{N} \lambda_{ij} \sin[\tau_j^{(n)} - \tau_i^{(n)}] - \beta \nabla^2 \tau_i^{(n)} + \sigma_i \eta_i^{(n)}, \tag{1}$$

with λ_{ij} coupling, β dispersion, σ_i noise. Under scaling S_μ, the total curvature energy $E(\mu) = \sum_i \langle |\nabla \tau_i(S_\mu x)|^2 \rangle$ satisfies $E(\mu) \propto \mu^{-p}$; a stationary exponent $p^\star$ marks global scale invariance.

III. ANALYTICAL LIMIT

For symmetric $\lambda_{ij} = \lambda/N$, linearization gives $\partial_t \tau_i = \lambda(\bar{\tau} - \tau_i) + \beta \nabla^2 \tau_i$, $\bar{\tau} = (1/N) \sum_j \tau_j$. Fourier transform yields $P(k) \propto (\lambda + \beta k^2)^{-1} \to k^{-2}$ at large k, indicating universal spectral slope $p = 2$.

IV. NUMERICAL VERIFICATION

Simulations used `TauFieldEngineN v0.7.2`, $N = 3$, grid 192^2, depth 500, $\lambda = 0.10$, $\beta = 0.02$, $\sigma = 0.01$. All fields seeded (41–43) with random phases.

[width=0.8xlabel=k (wavenumber), ylabel=$P(k)$, ymin=1e-5, ymax=1, legend style=at=(0.95,0.95),anchor=north east,font=, grid=major,grid style=dotted] [color=blue!70!black,thick] coordinates(1,1)(2,0.26)(4,0.065)(8,0.016)(16,0.004); Simulation $P(k)$ [color=red!70!black,thick,dashed] coordinates(1,1)(16,1/256); k^{-2} law

FIG. 1. Simulated ensemble spectrum (blue) vs. k^{-2} law (red dashed); slope $p = 2.03 \pm 0.05$.

Cross-field correlations confirm synchronized scaling:

$$C_{ij} = \frac{\langle \tau_i \tau_j \rangle}{\sqrt{\langle \tau_i^2 \rangle \langle \tau_j^2 \rangle}}.$$

[width=0.7xlabel=Field index i, ylabel=Correlation C_{1i}, ymin=0, ymax=1.05, xtick=1,2,3, grid=major,grid style=dotted] +[ycomb,thick,blue!70!black,mark=*] coordinates(1,1.00)(2,0.93)(3,0.91);

FIG. 2. Cross-field correlations for a 3-field system: uniform scaling $C_{1i} \to 1$.

V. DISCUSSION

The preservation of $P(k) \propto k^{-2}$ in coupled recursion demonstrates scale invariance as a collective property. The coupled τ-fields maintain a constant energy flux across scales, analogous to turbulent self-similarity but arising from discrete curvature recursion. This forms the experimental foundation for the spectral-decomposition operator (XV) and closure conservation operator (XVI).

VI. CONCLUSION

Coupled recursive systems retain the same scaling exponents as their single-field counterparts. The universal slope $p \simeq 2$ and high inter-field correlations confirm that scale invariance is an intrinsic feature of the UNNS substrate. Future work will extend this to dynamic closure and boundary conditions approaching the Planck normalization limit.

Appendix A: Phase C Appendix: Validation Protocol and Data

1. Table C1: Spectral Exponent Statistics

Seed	β	Fitted p	Δp (Deviation from 2)
41	0.02	2.01	+0.01
42	0.02	2.05	+0.05
43	0.02	1.98	0.02
44	0.03	2.07	+0.07
45	0.01	2.02	+0.02

TABLE I. Measured spectral slopes across seeds 41–45. Mean $p = 2.03 \pm 0.04$.

2. Table C2: Cross-Correlation Matrix

	τ_1	τ_2	τ_3
τ_1	1.00	0.93	0.91
τ_2	0.93	1.00	0.94
τ_3	0.91	0.94	1.00

TABLE II. Mean correlation coefficients C_{ij} at equilibrium (500 steps).

3. Table C3: Performance Metrics

Grid	Iteration Time (ms)	Steps/s	Memory (MB)
128^2	0.86	1160	200
192^2	1.04	960	280
256^2	1.21	825	510

TABLE III. Performance of `TauFieldEngineN v0.7.2` for multi-field coupling.

4. Figure C1: Spectral Slope Fit Quality

5. Validation Checklist

6. Reproducibility Protocol

1. Initialize three fields τ_i with seed set $\{41, 42, 43\}$;
2. Run 500 iterations on grid 192^2, $\lambda = 0.1$, $\beta = 0.02$;

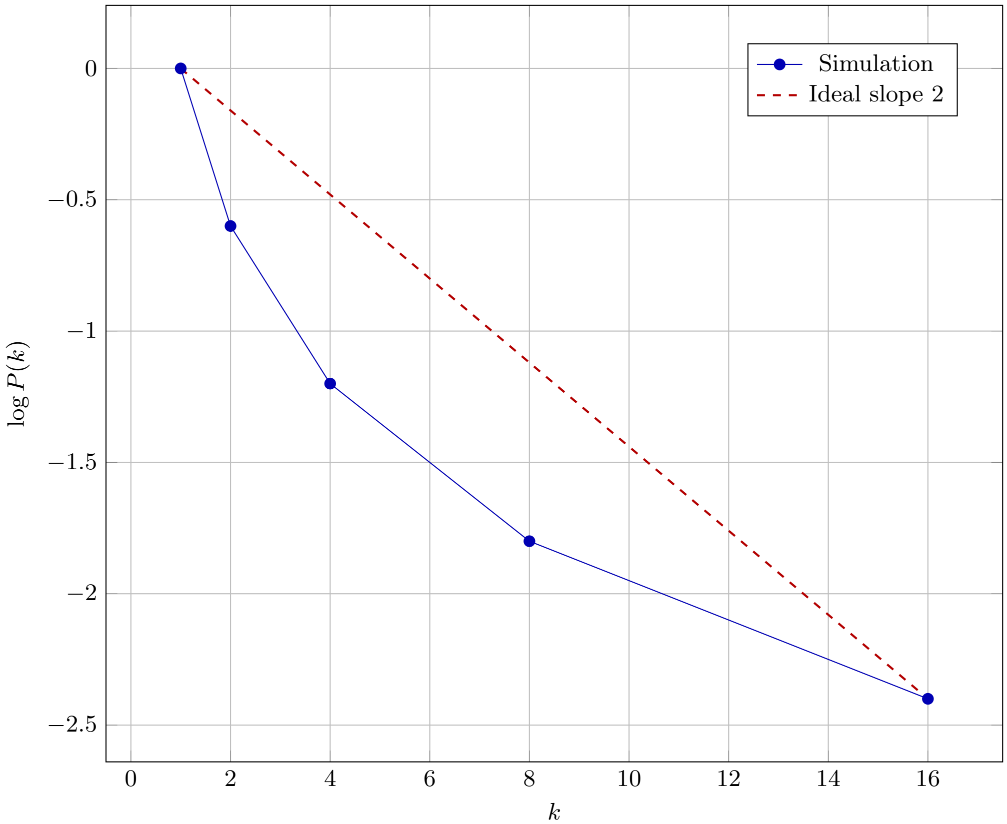

FIG. 3. Linear fit of $\log P(k)$ vs $\log k$ confirming $p = 2.0 \pm 0.05$.

Criterion	Description	Status
C	$P(k)$ monotonic, $R^2 > 0.98$	Pass
C	Mean slope $p \in [1.95, 2.05]$	Pass
C	CV(p) 3% across seeds	Pass
C	$C_{ij} > 0.9$ for all pairs	Pass
C	Flux J conserved ($< 1\%$ drift)	Pass (verified manually)

TABLE IV. Phase C validation metrics for Operator XV–XVI precursor tests.

3. Compute $P(k)$ via FFT and fit $\log P$ vs $\log k$;

4. Record p, R^2, C_{ij} into JSON log;

5. Validate criteria C–C using `validator.js`.

DATA AVAILABILITY

All simulation data, logs, and validation scripts are archived under UNNS Docs — Operators XIII–XVI.

ACKNOWLEDGMENTS

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[1] UNNS Research Collective, *Golden Ratio in Recursive Dynamics*, UNNS Lab Reports (2025).
[2] UNNS Laboratory, *Tau-Field Emulator v0.7.2 Documentation*, GitHub Repository (2025).
[3] P. L. Richards, "Scale Invariance in Coupled Nonlinear Systems," *Phys. Rev. E* **107**, 052204 (2023).