UNNS Operators XIV–XVI (Phase A): Definitions, Invariants, and Proof Sketches

UNNS Research Collective (2025)

Unbounded Nested Number Sequences Laboratory (UNNS)

OVERVIEW

Phase A formalizes the theoretical cores of the next operators in the UNNS hierarchy: XIV (Φ-*Scale*), XV (*Prism*), and XVI (*Closure*). We provide precise operator definitions, invariants, and proof sketches establishing well-posedness, existence of measurable observables, and inter-operator consistency with Operator XIII (*Interlace*).

I. PRELIMINARIES

Let $\tau^{(n)} : \Omega \subset \mathbb{R}^d \to \mathbb{R}$ be the τ-Field at recursion step n. We use:

$$\kappa^{(n)}(x) = \nabla \cdot \big(\nabla \tau^{(n)}(x)\big), \qquad H_r^{(n)} = \text{Shannon entropy of a suitable observable of } \tau^{(n)}.$$

The Interlace lock (Operator XIII) fixes a phase ratio with parameter $\lambda^\star$. Throughout, η_σ denotes mean-zero noise with variance σ^2.

II. OPERATOR XIV — Φ-SCALE (RECURSIVE PROPORTION)

A. Definition

Let $\mathcal{S}_\mu : \Omega \to \Omega$ be a scaling map $x \mapsto \mu x$ (with suitable boundary handling). Define the scale-coupled update

$$\tau^{(n+1)}(x) = \tau^{(n)}(x) + \lambda \sin\!\big(\tau^{(n)}(\mathcal{S}_\mu x) - \tau^{(n)}(x)\big) + \eta_\sigma. \tag{1}$$

Define

$$\Delta^{(n)}_{\text{scale}}(\mu) = \Big\langle \big(\tau^{(n)}(\mu x) - \tau^{(n)}(x)\big)^2 \Big\rangle_x, \qquad \Pi^{(n)}(\mu) = \Big\langle \cos\big(\tau^{(n)}(\mu x) - \tau^{(n)}(x)\big) \Big\rangle_x.$$

Let $S^{(n)}(k)$ be the power spectrum of $\tau^{(n)}$.

B. Invariants and Lock

Invariant (self-similar spectrum). On a scaling band $B \subset \mathbb{R}^+$,

$$S^{(n)}(k) \approx \mu^{-\gamma}\, S^{(n)}(\mu k) \quad \text{for some } \gamma > 0. \tag{2}$$

Scale lock. A *lock* occurs at $\mu^\star$ where $\Delta^{(n)}_{\text{scale}}(\mu)$ attains a sharp minimum and $\Pi^{(n)}(\mu)$ a local maximum, stable in n.

C. Proof Sketches

(i) Well-posedness. For $\tau^{(n)} \in L^2(\Omega)$ and bounded λ, the r.h.s. of (1) is measurable and square-integrable. Noise η_σ preserves L^2 in expectation. Hence $\tau^{(n)} \in L^2$ for all n.

(ii) Existence of $\mu^\star$. Consider $f(\mu) = \mathbb{E}[\Delta^{(n)}_{\text{scale}}(\mu)]$. Under mild regularity, f is continuous on compact μ-intervals. Therefore a minimizer exists. Uniqueness in practice follows from convexity near Interlace lock $\lambda^\star$ (empirically: a single sharp basin).

(iii) Self-similarity fit. If (1) induces correlation between x and μx with bounded mixing, the spectrum satisfies (2) up to a multiplicative law; γ emerges from regression of $\log S(k)$ on $\log S(\mu k)$.

III. OPERATOR XV — PRISM (SPECTRAL DECOMPOSITION)

A. Definition

Augment the recursion with a dispersive channel $\beta \geq 0$:

$$\tau^{(n+1)}(x) = \tau^{(n)}(x) + \lambda\, \sin(\Delta\phi^{(n)}(x)) - \beta\, \nabla^2 \tau^{(n)}(x) + \eta_\sigma, \tag{3}$$

where $\Delta\phi^{(n)}(x)$ denotes a phase difference functional (as in XIII/XIV). Define spatial spectrum $P^{(n)}(k) = \left|\widehat{\kappa^{(n)}}(k)\right|^2$ and temporal spectrum $P^{(n)}(\omega)$.

B. Spectral Invariants

Power-law band. There exist (k_1, k_2) with

$$P^{(n)}(k) \sim k^{-p} \quad \text{on } [k_1, k_2], \qquad p > 0. \tag{4}$$

Feature set. Persistent peaks/notches $\{k_i\}$ satisfy geometric relations predicted by $\mu^\star$ from XIV (near-log periodic spacing).

C. Proof Sketches

(i) Energy balance. The $-\beta\nabla^2$ term damps high-k modes; the $\sin(\cdot)$ term injects structure at phase-coupled scales. Under stationary statistics, a balance produces a scale band with power-law decay.

(ii) Imprint of XIV. If XIV establishes self-similarity with $\mu^\star$, then phase-coherent interactions seed harmonics at $\{k_i\} \approx \{k_0\mu^{\star m}\}$; these persist as spectral features in $P(k)$.

IV. OPERATOR XVI — CLOSURE (SEALING THE MANIFOLD)

A. Definition

Let $\{\tau^{(0)}, \ldots, \tau^{(N)}\}$ be a trajectory generated by XIII$\mapsto$XIV$\to$XV. Define an information-flux functional $\mathbf{J}$ and a sealing step

$$\tau^{(N+1)} = \tau^{(N)} - \alpha_c \nabla \cdot \mathbf{J}\big(\tau^{(0..N)}\big), \qquad \alpha_c > 0. \tag{5}$$

Let $\mathcal{M}$ be a collection of conserved metrics (e.g., integrated curvature, entropy bands, invariant tuples).

B. Closure Properties

Conservation. $\mathcal{M}\big(\tau^{(N+1)}\big) = \mathcal{M}\big(\tau^{(0)}\big)$ up to tolerance ϵ_c.

Idempotence. $\mathcal{C}(\mathcal{C}(\tau)) = \mathcal{C}(\tau)$, where $\mathcal{C}$ applies (5) with the same $\mathbf{J}$ and α_c.

C. Proof Sketches

(i) Existence of sealed state. Define $\mathcal{F}(\tau) = \tau - \alpha_c\nabla \cdot \mathbf{J}$. If $\mathcal{F}$ is a contraction in a norm consistent with $\mathcal{M}$, Banach's fixed-point theorem yields a unique sealed state.

(ii) Idempotence. If the sealed state satisfies $\nabla \cdot \mathbf{J} = 0$, then reapplication of (5) produces no change, establishing idempotence.

V. INTER-OPERATOR CONSISTENCY

Running XIV at $\lambda^\star$ (from XIII) sharpens the scale-lock basin (unique $\mu^\star$). XV inherits the spacing pattern from $\mu^\star$ in $P(k)$. XVI preserves the invariant tuple $\{\sin^2\theta_W, \mu^\star, p\}$ within specified tolerances while minimizing net flux.

PHASE A DELIVERABLES

- Formal definitions (1), (3), (5) with measurable invariants.
- Existence/well-posedness sketches; lock/self-similarity arguments.
- A shared invariant tuple for cross-operator consistency.